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A QUASI-STATIC DESIGN MODEL FOR SYNTHETIC MARINE TOWLINES

by

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Submitted to the Department of
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ABSTRACT

A model of the elongation characteristics of a double braid nylon rope is presented as an aid to sizing a marine towline. The material and structural elongation properties are combined and modeled as being the sum of a non-elastic permanent elongation and a load dependent elastic elongation. The loading range of practical interest is examined and a design factor of safety, with both an upper and lower bound, is established. The lower bound is incorporated to limit relative movement between structural elements in an attempt to control internal abrasion. The upper bound is imposed to stay within the safe working load and total elongation limits of the rope. The governing equations for the towline are then solved within these constraints for a typical submarine towing system to provide a towline capable of sustaining the hydrodynamic loads due to the resistances of the submarine and the towline. It is shown that the resistance of a typical towline may be of the same order of magnitude as the resistance of the towed vessel and cannot be neglected. The solution results in a recommended towline diameter and breaking strength which is then examined under possible loading conditions other than those selected for the initial design. It is shown that a significant portion of the operating envelope is outside the limits obtained from the factor of safety analysis and that a nylon towline may be vulnerable to significant internal abrasion, especially if the line is oversized. Alternate materials are discussed but not analyzed due to the lack of experimental data which could support a functional model of elastic elongation behavior.

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LIST OF SYMBOLS

$A1$	- Empirically determined constant
A_0	- New-wet cross-sectional area of rope at T_0
A_w	- Wet cross-sectional area of towline at T_w
B	- Buoyancy of towline per unit length
B_s	- Average breaking strength
B_m	- Minimum guaranteed breaking strength
b	- Beam of towed vessel
C_n or CD	- Normal drag coefficient of towline
C_t	- Tangential drag coefficient of towline
C	- Arbitrary constant
d_0	- New-dry diameter of rope at T_0
d_w	- Diameter of towline at T_w
d	- Diameter of rope under arbitrary load T
ds_0	- New-wet elemental length at T_0
ds_w	- Wet working elemental length at T_w
ds	- Elemental length under arbitrary load T
DNF	- Dry-Nylon elastic elongation Function
DNC	- Dry-Nylon design curve
e	- Elemental elastic elongation under arbitrary load T
$e_r=\Delta L_r/L_0$	- Residual strain
$e_h=\Delta L_h/L_r$	- Hysteresis strain
$e_w=\Delta L_w/L_w$	- Working strain
$e_n=\Delta L_n/L_0=(\Delta L_r+\Delta 1_n)L_0$	- Non-elastic strain
$e_e=\Delta L_w/L_0$	- Elastic strain
$e_t=(\Delta L_r+\Delta L_n+\Delta L_w)/L_0$	- Total strain
FS	- Factor of safety (design factor)
F_t	- Tangential drag per unit length of towline

F_n	- Normal drag per unit length of towline
F_{cs}	- Control surface force
g	- Gravitational constant
Kap	- Apparent spring constant of towline
Kaps	- Specific apparent spring constant of towline
l	- Length of towed vessel
L_0	- Base length - length of new, dry, unused rope measured at the base load
L_w	- Working length - length of rope measured at base load immediately after fifty cycles of loading to the working load. The working length can be effected by the conditions of use and for each case the wet or dry condition should be specified
L_r	- Recovered length - length of the rope measured at the base load after the rope has been subjected to fifty cycles at the working load, is completely unloaded and left in a relaxed state for thirty minutes, and then reloaded to T_0
m	- Empirically determined exponent
MFSN	- Marine factor of safety for double braid nylon rope
PI	- Constant, 3.14159
R_t	- Resistance of towed vessel in lbs.
S	- Wet length of towline under arbitrary load
T	- Arbitrary tension in towline
T_{ah}	- Horizontal tension component at towed vessel
T_{av}	- Vertical tension component at towed vessel
TD	- Depth of towed vessel
T_0	- Base load = 200 d^2 lbs (d is the new rope diameter in inches)
T_w	- Working load, Lb - a load representative of the application. For standard tests 20% of the rope average breaking strength is used

U - Velocity of tow system

v - Velocity of local current

V - Relative velocity of towline with respect to the water

V_n - Normal component of relative velocity

V_t - Tangential component of relative velocity

W - Towline weight per unit length

x - Horizontal coordinate

z - Vertical (depth) coordinate

ΔL_r - Residual elongation - The portion of elongation which is not recoverable (also called Permanent Elongation)

ΔL_h - Hysteresis - That portion of the elongation which is recovered over a period of time. Note: this is not the hysteresis loop, but is a definition adopted by the cordage industry

ΔL_w - Working elongation - That portion of the elongation which immediately recovers when load is removed (also called Elastic Elongation)

ΔL_n - Non-elastic elongation - The amount of extension which exists when load is removed but no time is given for hysteresis recovery. It is the sum of residual and hysteresis elongation

ΔL_t - Total stretch - The entire length change in a rope when placed under a given load (cyclic or otherwise), and includes the residual, hysteresis and working elongations

Φ - Local angle of towline with respect to the horizontal

τ - Specific tension (T/B_s)

ρ_ℓ - Mass density of towline, 2.209 slugs/ft^3 for nylon

ρ_w - Mass density of sea water, 2 slugs/ft^3

Chapter I

INTRODUCTION

A current problem in ocean engineering is the incorporation of synthetic rope into lifting, handling, mooring, and towing systems. The high strength-to-weight ratios, the energy absorption characteristics, and the flexibility and ease in handling are all features offered by synthetic rope that are not common to wire rope or natural fiber rope. The adaptation of these characteristics to marine engineering problems can result in improved system safety and performance if the design is well executed. The design of an efficient ship-to-ship towing system requires an analysis which brings together those material and structural characteristics needed to sustain the static and dynamic loads imposed by the connected vessels as they respond to the excitation of the ocean environment.

One of the most important design elements of the towing system is the ability to absorb dynamic loads and dissipate energy without damaging system components or degrading system performance. This can be accomplished through employment of mechanical tensioning devices, steel/chain catenaries, synthetic towlines and, in some special cases, the use of the control surfaces on the towed vessel. The application of a synthetic towline is the most universal of these methods as it is easily adapted to both scheduled and emergency tows. The resort to synthetic rope on such occasions has been common for several years, but has recently been discouraged, and even forbidden by the U. S. Navy in scheduled military tows, and by Lloyds of London for insured commercial tows. Both of these actions were apparently the result of the unacceptably high failure rate of synthetic towlines and the resulting damage. There are <u>many</u> possible causes of towline failures. While it is not the intent of this thesis to address them all, we submit that failure rates can be reduced and conditions leading to failure can be more accurately predicted by improved design processes.

This thesis will address the incorporation of synthetic rope into the marine towing system. Nylon, polyester, polypropylene, and Kevlar are the four primary fibers used to make synthetic rope, each with distinct mechanical characteristics capable of improving the system being designed. However, such variations in characteristics also require that slightly different methods be used in the analysis. Nylon will be used here as the base material for the presentation because it encompasses the vast majority of the ropes presently in use. Even though the methods to be suggested will be adaptable to other materials, it will be imperative that differences in mechanical characteristics be accounted for. In addition, the structure of the rope will be limited to 2-in-1 double braid since it is a torque-free structure and offers the highest breaking strength for a given material and size when compared to the other common commercial rope structures.

This design process will present a method of predicting the elastic behavior of a nylon towline, examine the effects of the design "factor of safety", present a quasi-static preliminary design and examine the sensitivity of the design to each of the design parameters. This process of examining the interaction between material properties, loading conditions, and design methodology will lead to the preliminary sizing of the synthetic rope and to subsequent improvements of the tow system through trade-offs between design parameters.

Chapter II

ELONGATION CHARACTERISTICS

2.1 Background

To describe the elastic elongation characteristics of synthetic ropes requires the selection of the best working approximation to a complex mechanical structure comprised of a very large number of basic filament elements. Polymeric fiber elements are, in fact, viscoelastic in nature; any approximations that ignore time dependence are inherently inexact. Polymeric fibers subjected to a sustained load, such as the mean towing load, are known to creep. This creep is reflected in an increase in nonrecoverable elongation and the fiber will eventually rupture; this creep rupture is a function of load amplitude and time under load. A fatigue failure model has been developed for predicting failure of low twist yarns based on creep rupture data from fibers [11], but this model has not yet been extended to marine rope level. The inclusion of the time factor in a truly viscoelastic model which could be matched with the complex time dependent excitation forces of the ocean environment cannot be done at this time.

The task is further complicated by the need to design towing systems requiring the use of large ropes which have not been extensively tested. Comparison of data which have been collected by various sources must be undertaken with caution [4], since test methods, environmental conditions, and specimen size and geometry greatly influence results obtained. Several ongoing development programs [8] are underway, focusing on various aspects of synthetic fiber rope behavior in the marine environment. When such programs have been completed, it should be possible to eliminate many of the unknowns that now hinder the design process. In the interim some uncertainties must be accommodated in the design methods used. One such method is presented here.

2.2 Load-Elongation Characteristics

It is essential that the elongation characteristics peculiar to synthetic ropes be understood at the outset. An introduction to the subject is here provided with the aid of Fig. 1. A more complete description can be found in [1, 2, and 3].

As a synthetic rope is loaded for the first time from T_0 to a working load T_w, the load-elongation curve follows the path A-B of Fig. 1(a). If the load is then decreased to T_0, the second portion of the curve B-C is traced; as the load is again increased to T_w, the path C-D is formed and the typical hysteresis loop can be seen. The distance A-C represents the non-elastic elongation, ΔL_n, part of which would be recovered if the rope were allowed to relax for extended periods between cycles. Subsequent cycles will cause the hysteresis loop to migrate to the right as the non-elastic elongation is increased with each load cycle. The hysteresis loop "stabilizes" [1] at approximately fifty cycles and it is this loop that is used for design applications that require cyclic loading.

Figure 1(b) shows the first, tenth, and fiftieth cycles of a typical rope stabilization profile [1] with the pertinent components of the total elongation labeled. It can be seen that the total elongation is composed of the non-elastic elongation and the working elongation. These elongations can be converted to strains by dividing by a "reference" length and at this point extreme care should be taken when comparing data from different sources because the same "reference" length is not always used. For example, reference [1] defines working strain as the change in working length over the working length, where the working length is the base length L_0 plus the non-elastic elongation ΔL_n. In contrast, reference [2] defines the working strain as the elastic change in length over the base length. The relation is:

$$e_e = e_w \times L_w/L_0 \qquad (1)$$

For nylon working between twenty and thirty percent of the

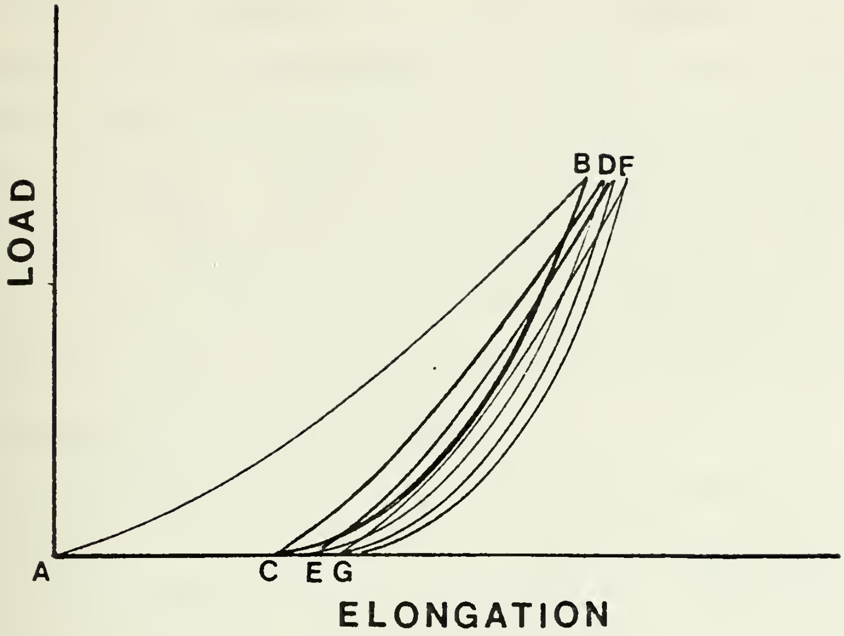

FIGURE 1a

TYPICAL LOAD-ELONGATION BEHAVIOR OF NEW AND UNUSED ROPES

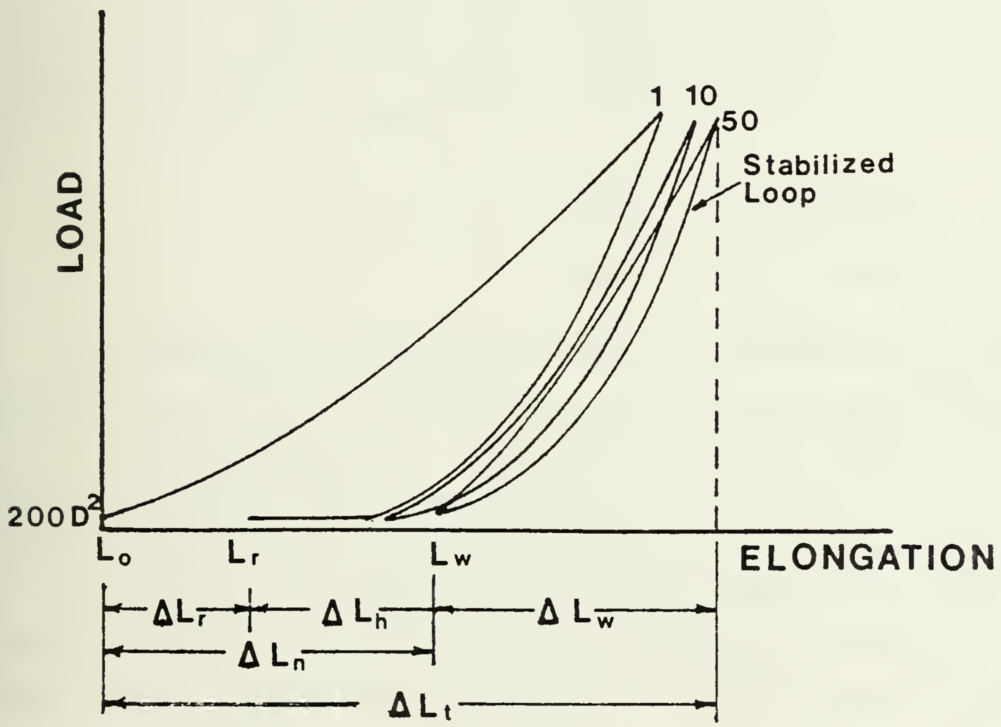

FIGURE 1b

TYPICAL ROPE STABILIZATION PROFILE

average breaking strength, B_s, L_W/L_0 is approximately 1.13 [1]. Another contrast is found in Reference [4], for the testing of wet single-point-mooring hawsers, where the reference length was measured at T_0 with the rope wet. In this case, a direct conversion is not possible for nylon because of the shrinkage which causes the fiber to swell and the length to shorten. However, the author [4] states that the results of a few tests were re-analyzed and that the elasticity reported was about 1% above that based on the working length.

2.3 Elastic-Elongation Model

With these definitions it is proposed [1] that rope size can be eliminated from the stress-strain relation by dividing the actual tension, T, by the average rated breaking strength, B_s, to produce the specific tension, τ, which can then be related to the working strain by a function of the form:

$$\tau = A1 \times (e_W)^m \qquad (2)$$

where A1 and m are empirically determined constants and the rope has been stabilized with 50 load cycles at 20% of the breaking strength. For nylon in both the wet and dry condition, the function becomes:

$$\tau = 9.78 \times (e_W)^{1.93} \text{ (wet nylon)} \qquad (3)$$

$$\tau = 14.2 \times (e_W)^{1.71} \text{ (dry nylon)} \qquad (4)$$

In addition to these functions, Reference [5] presents a design curve for a 2.5-inch double braid rope subjected to cyclic load conditions. Again, the design curve is based on data collected during the fifty-first cycle at 20% of B_s. Reference [6] presents a "linearized (retention) equation" for nylon towlines which is size dependent; this function is presented as part of a towing system design report and the basis for the equation is not given. This report uses E for the elastic elongation of the towline in feet, and is:

$$E = (T \times L)/(7.4 \times 10^4 \times d^2) \qquad (5)$$

Since E is stated to be the elastic elongation, it is

assumed to be equivalent to ΔL_W. By rearranging the terms and dividing the tension with the rated breaking strength, to produce a function of specific tension for the given size rope, the equation can be compared with the above functions in the form:

$$E/L_0 = (T/B_s)/[(7.4 \times 10^4 \times d^2)/B_s] \quad (6)$$

where it has been assumed that the length intended was the base length, L_0. The results of this equation can now be plotted for ropes having diameters in the range of interest and compared to the size independent functions, equations (3) and (4).

In Fig. 2, a common "reference" length of L_0 has been established and for the functions given by McKenna [1], the data given by Flory [4], the typical design curve presented by Wong [5], and the linear function used in the towing system report [6], the respective elastic elongation curves are presented for approximate comparison. In Fig. 2a, curve (1) represents the wet nylon function, WNF, and curve (2) represents the dry nylon function, DNF, both of which are size independent, but based on data taken after the fiftieth load cycle to 20% B_s. Curve (3) represents the design curve, DNC, given by Wong for 2.5-inch diameter dry double braid nylon rope on the fifty-first cycle. Curves (L1.75) and (L 2.5) represent the "linearized (retention) equation" for 1.75- and 2.5-inch diameter ropes. It appears that the linear function is fairly accurate at low specific loads when compared to the dry nylon function and the dry nylon curve. In Fig. 2b, curves (4) through (9) represent data taken at 10, 100, 300, 1,000, 3,000, and 10,000 cycles [4] for 1.75-inch diameter pre-soaked double braid nylon ropes. These curves for pre-soaked double braid nylon ropes more closely follow the dry nylon function. The reaction of the pre-soaked ropes can be partially justified by assuming that the water is being squeezed out as the rope is cycled under load which results in increased friction between structural elements. This would probably produce only small changes in the elastic

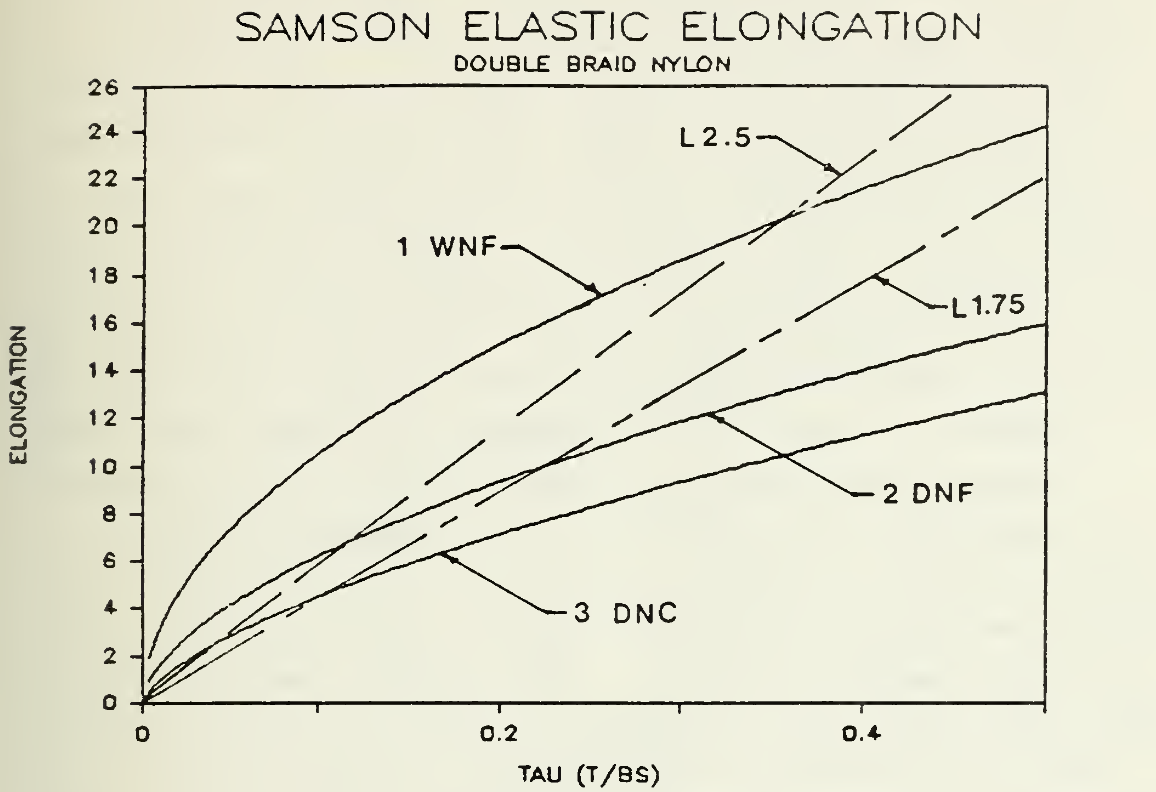

FIGURE 2a

PLOT OF ELASTIC ELONGATION DESIGN MODELS FOR DOUBLE BRAID NYLON ROPE

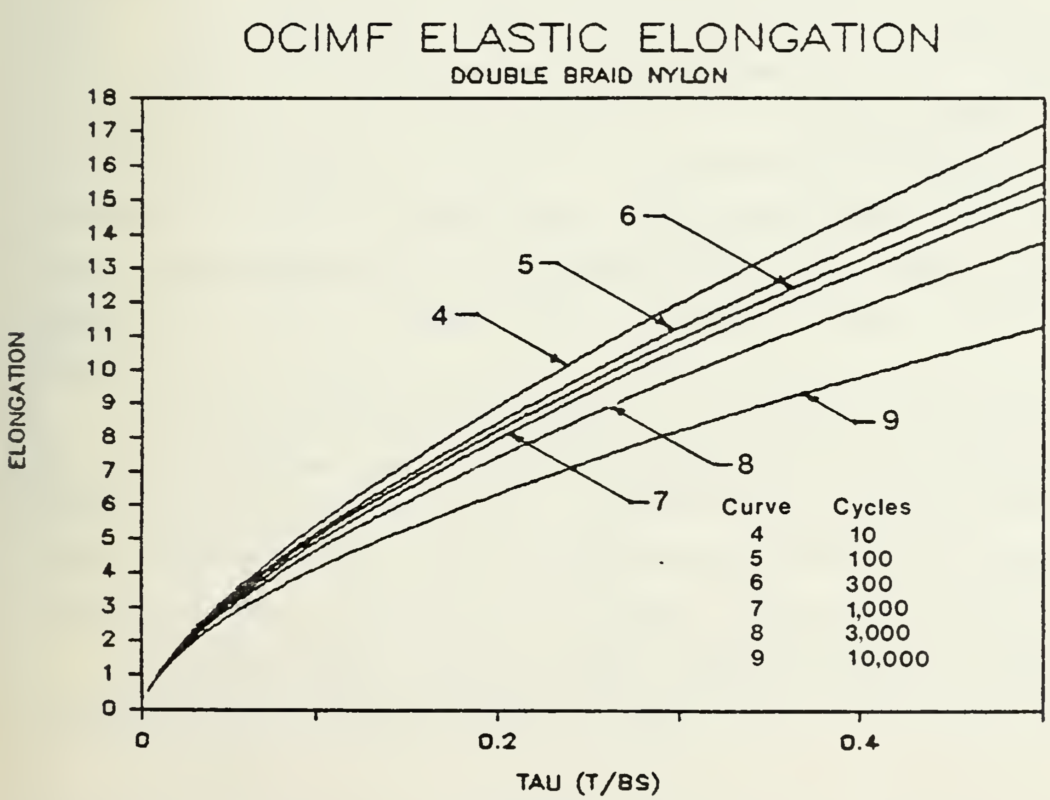

FIGURE 2b

PLOT OF CYCLIC ELASTIC ELONGATION DATA FOR DOUBLE BRAID NYLON ROPE

behavior of the rope. It can also be assumed that the rope was beginning to dry out [4] from internal heat generation and exposure to the air during testing; approximately 2,500 cycles were applied each day [4]. Both of these factors would tend to shift the curves toward the dry nylon model, as they are shown in Figs. 2a and 3. In Fig. 3, curve (3) was eliminated because of the uncertainty in the original reference length and only the curves (5) and (8), representing 100 and 3,000 cycles, were retained from the pre-soaked cyclic load tests. Once again these are compared to the linear functions presently used to design some U. S. Navy tow systems.

Figure 4 presents the non-dimensional stiffness, i.e. the change in percent load with a change in percent elongation, and it is here that the validity of the linear approximations becomes questionable. If it is assumed that the curve (1) for wet nylon is closest to the actual rope behavior, then the linear approximation is valid only in the region of the 15% load range and that there is a deviation of between 30% and 50% when the linear function is compared to the wet nylon function, curve (1), for a cyclic load amplitude about the mean load of plus or minus 10%, as is evident when the respective values are compared at τ equal to 5% and 25%. Such a load amplitude could reasonably be expected under working conditions. In contrast, if it is assumed that the dry nylon function, curve (2), or the 100 and 3,000 cycle curves (5) and (8) approximate the real condition, then the linear approximation is accurate only very near a specific load of 5%, which should not represent normal working conditions.

Based on these comparisons no further consideration will be given to the linear approximation of the load-elongation behavior of nylon towlines, since it is apparent that both the elongation and the stiffness are very non-linear below 20% of B_s and it is in this region that we are most interested.

To this point, it has been assumed that a rope is completely stabilized by the fiftieth load cycle and that

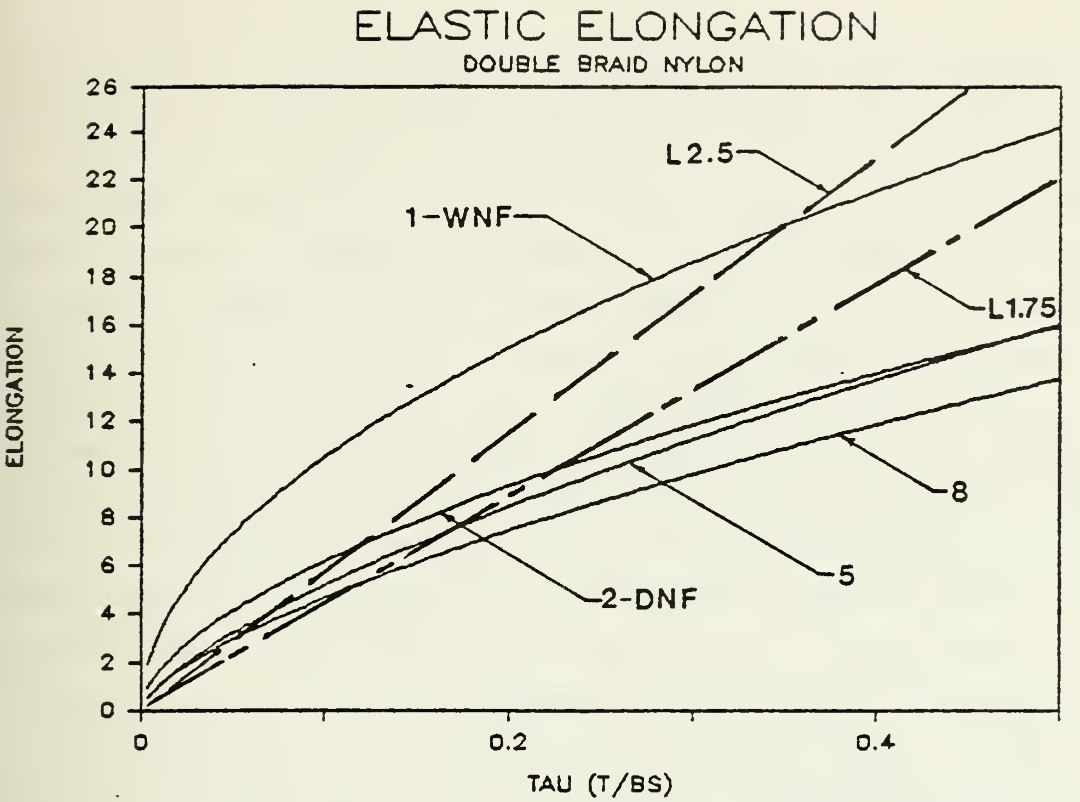

FIGURE 3

COMPARISON OF ELASTIC ELONGATION DESIGN MODELS WITH DATA FOR DOUBLE BRAID NYLON ROPE

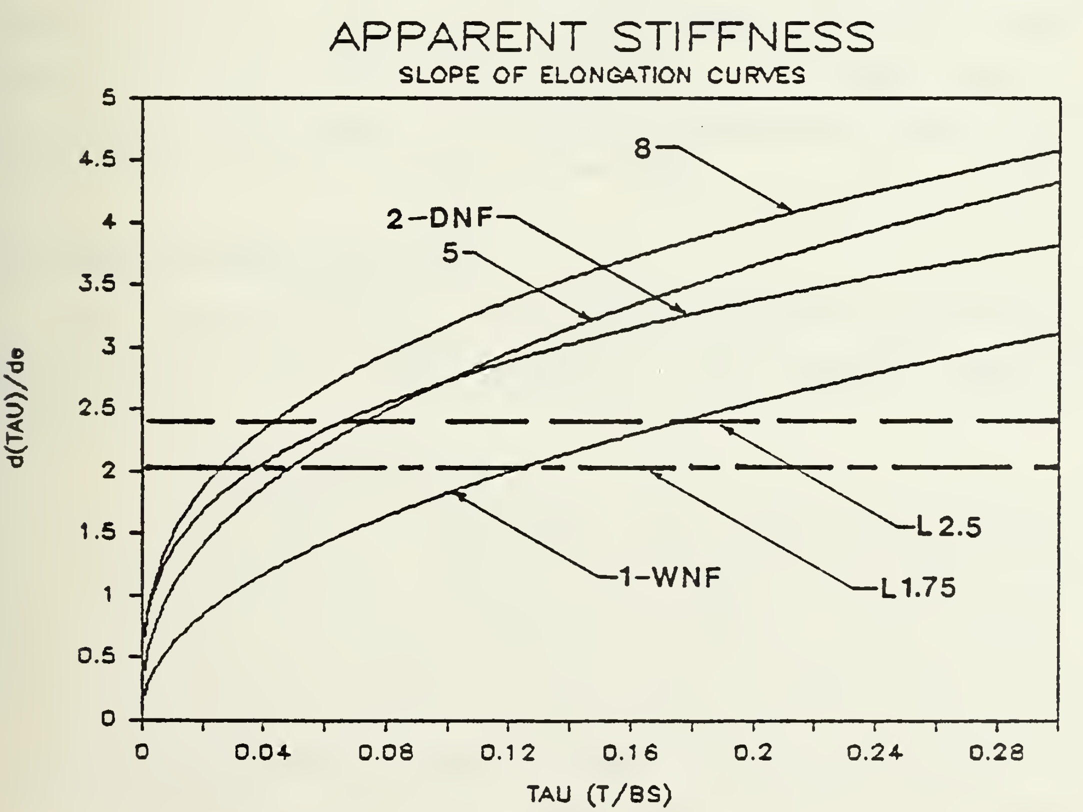

FIGURE 4

APPARENT STIFFNESS OF DOUBLE BRAID NYLON ROPE

further creep migration of the hysteresis loop will not be significant, except in the case where the rope is unloaded and allowed to recover slowly for an extended period, in which case the rope may need to be "broken-in" again. The fiftieth cycle stability assumption is the basis for Eqs. (3) and (4) and for the proposed design curve [5], curve (3). In fact, the assumption is based on synthetic rope test standards [7]. This is not a true representation for an extremely high number of load cycles that could be expected during a towing operation (5,000 to 15,000 cycles per day) which may encompass several days of continuous operations where there is a variable but constantly applied mean load. The hysteresis loop actually has a nearly logarithmic migration [1], with the exceptions noted above, and the permanent elongation will continue to increase as the elastic elongation decreases. This is simply the manifestation of the viscoelastic properties of the nylon filaments and even though it cannot be quantified in the complex structure of a large rope, it must be accounted for. Referring to Fig. 2a, it can be seen that such a migration would cause the high cycle wet nylon curve (1) to shift again toward the low cycle dry nylon curve (2). Similar to the shift seen in Fig. 2b, as the number of cycles is increased from 10 cycles, curve (4) to 10,000 cycles curve (9).

2.4 Comparison with Data

For these reasons, the elastic elongation function for dry nylon, Eq. (4), will be used in this thesis, recognizing that it is:

(1) Approximate,

(2) Chosen to represent high cycle wet behavior.

Under conditions where a new towline was being installed for a specific task, it might be prudent to use the wet nylon function to predict initial load elongation behavior, providing a lower bound on load amplification and an upper bound on elongation. Then, to use the dry nylon curve to provide the upper bound on load amplification and lower bound

on elongation, the condition that could be expected after the towline had undergone a significant number of cycles.

The basic assumption thus far has been that the load elongation behavior can be normalized by dividing by the average rated breaking strength to eliminate the effects of changes in the size of the rope. This assumption was checked by comparing the predicted stiffness, using the function shown in Fig. 4, with an apparent spring constant, Kap, presented by Bitting [3]. This was done by dividing the apparent spring constant by the manufacturers published breaking strength to produce a specific apparent spring constant which is a function of frequency, f, mean load, T_m, and load amplitude, DT:

$$\mathrm{Kaps} = \mathrm{Kap}/B_s \tag{7}$$

Bitting presents coefficients to be used in a Box-Behnken equation (7a) for the apparent spring constant for ½", 3/4", 1", and 1¼" double braid nylon line:

$$\begin{aligned} \mathrm{Kap} = {} & B1 + B2 \times f^2 + B3 \times Tm^2 + B4 \times DT^2 + \\ & + B6 \times Tm + B7 \times DT + B8 \times f \times Tm + B9 \times \\ & \times f \times DT + B10 \times Tm \times DT \end{aligned} \tag{7a}$$

All of these coefficients were obtained by testing the ropes in the wet condition and are considered to be valid only in the range of testing [3], which is for a specific load greater than 10%. The comparison is shown in Fig. 5, where it is seen that in the 20% load range the dry nylon function provides a good model for Bitting's ½", 1", and 1¼" data. The departure of the 3/4" apparent spring constant cannot be explained at this time. Based on the relative magnitude of the coefficients in Eq. (7a), it is found that the apparent spring constant, Kap, is primarily a function of mean load, Tm, and load amplitude, DT. The frequency of loading, f, has a minimal effect within the range of interest. In Fig. 5a the maximum permissible frequency and load amplitude as given by Bitting were used to plot the curves, while in Fig.

COMPARISON OF DESIGN MODELS AND APPARENT SPRING CONSTANT AS REPORTED BY BITTING

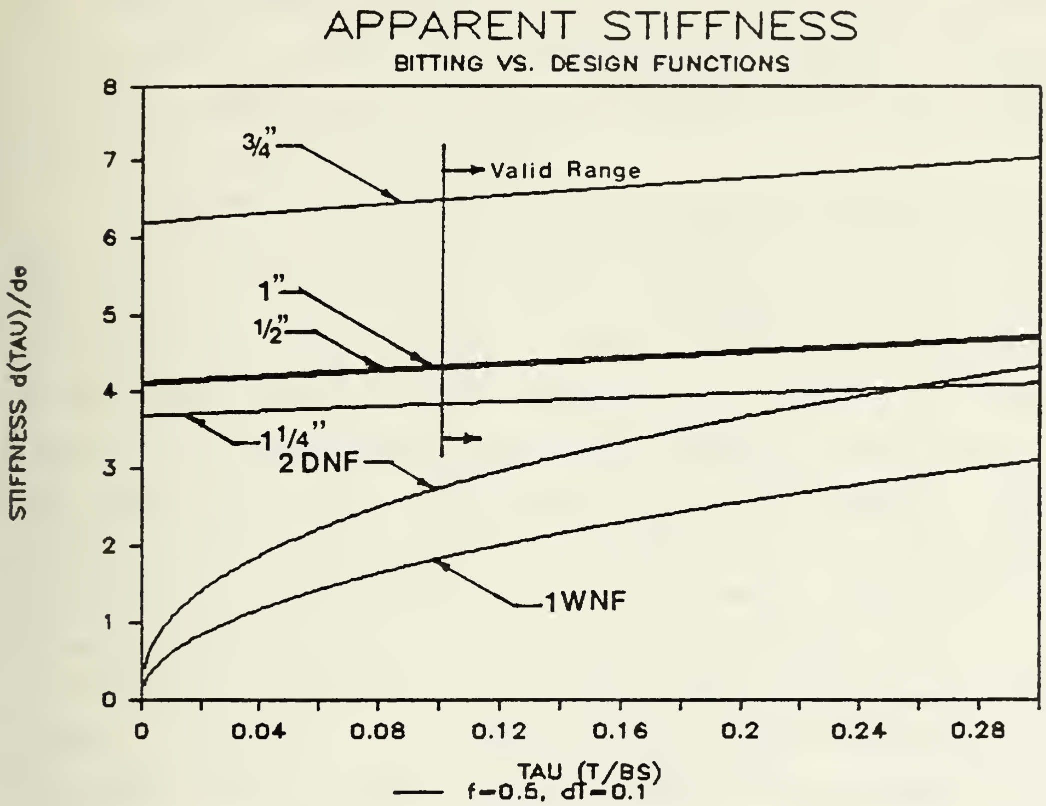

FIGURE 5a

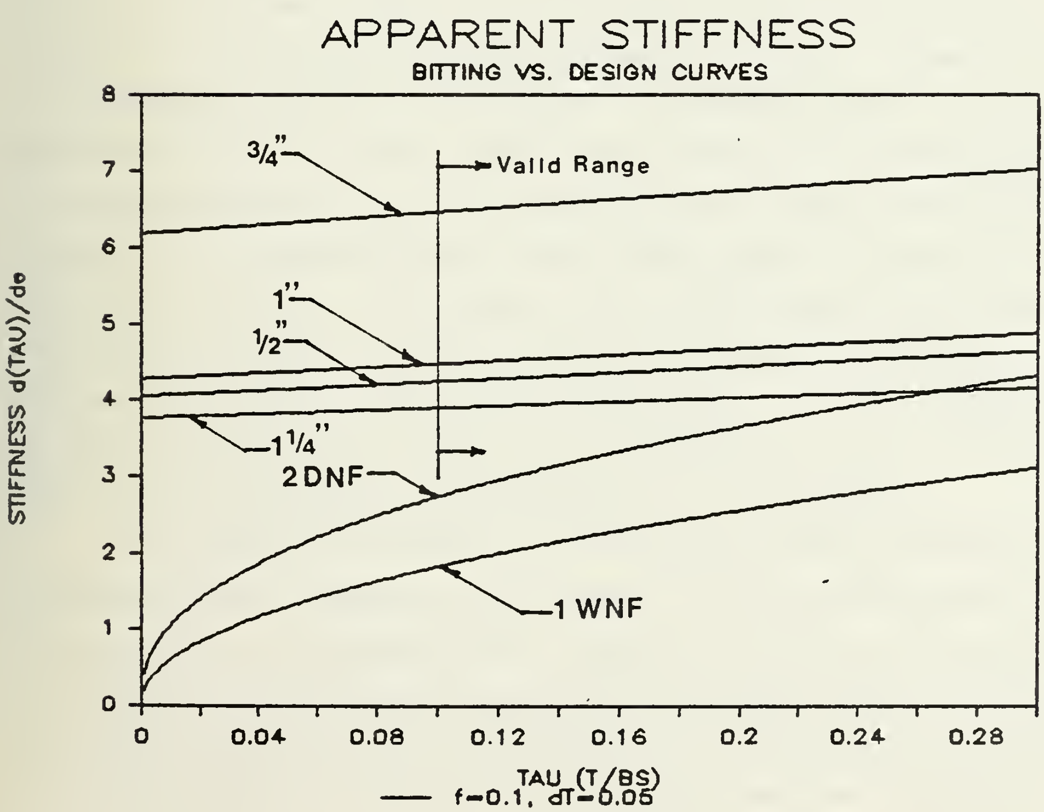

FIGURE 5b

5b, the minimum permissible values, also given by Bitting, were used. A comparison of these two figures reflects the minimal effect of varying the frequency and load amplitude over the total valid range.

Further load elongation data were presented by Gibson and Wolfe [12] for 4½" circumference double braid nylon rope tested in both the dry and wet condition. These data are presented in Figs. 6 and 7, where the wet and dry nylon functions have been added for comparison. It is interesting to note that these curves represent tensile tests where the specimens had been conditioned with 250 cycles to the base reference load, T_0, and then loaded in a tensile break test. This base reference load is a very small percentage of the breaking strength, but the elastic elongation in the lower load ranges remains in good agreement with the proposed model. From Fig. 6, it can be seen that the dry tensile specimen falls between the dry and wet functional models; further, the test results are very close to the dry-nylon functional model throughout the range of practical working loads. The wet tensile specimen shown in Fig. 7 is very close to the wet-nylon functional model in the lower load range and indicates good agreement with the lower bound stiffness referred to earlier. A second point that must be mentioned here is that the ropes tested by Gibson and Wolfe were manufactured by Wall Rope Works; this is significant in that all of the previous data were taken on ropes manufactured by Samson, and it was believed that there could be variations between different manufacturing companies. This concern may still be valid, but based on this admittedly limited comparison, the variations may not be a major factor in the preliminary sizing phase of the design.

It should be noted at this point that polyester and polypropylene reaction with water at the molecular level is negligible, at temperatures commonly found in the ocean environment, and thus do not exhibit significant shrinkage or strength reduction when exposed to the marine environment. A single elastic elongation function may be sufficient to

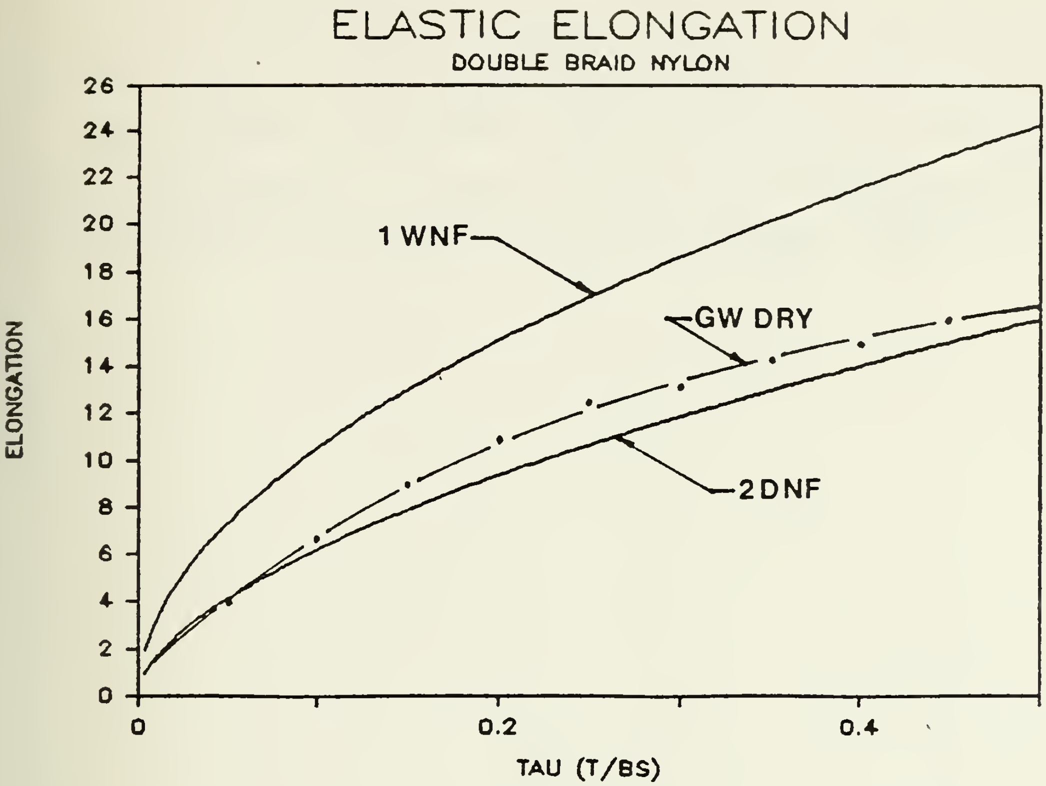

FIGURE 6

COMPARISON OF DESIGN MODELS WITH DRY ROPE DATA REPORTED BY GIBSON AND WOLFE

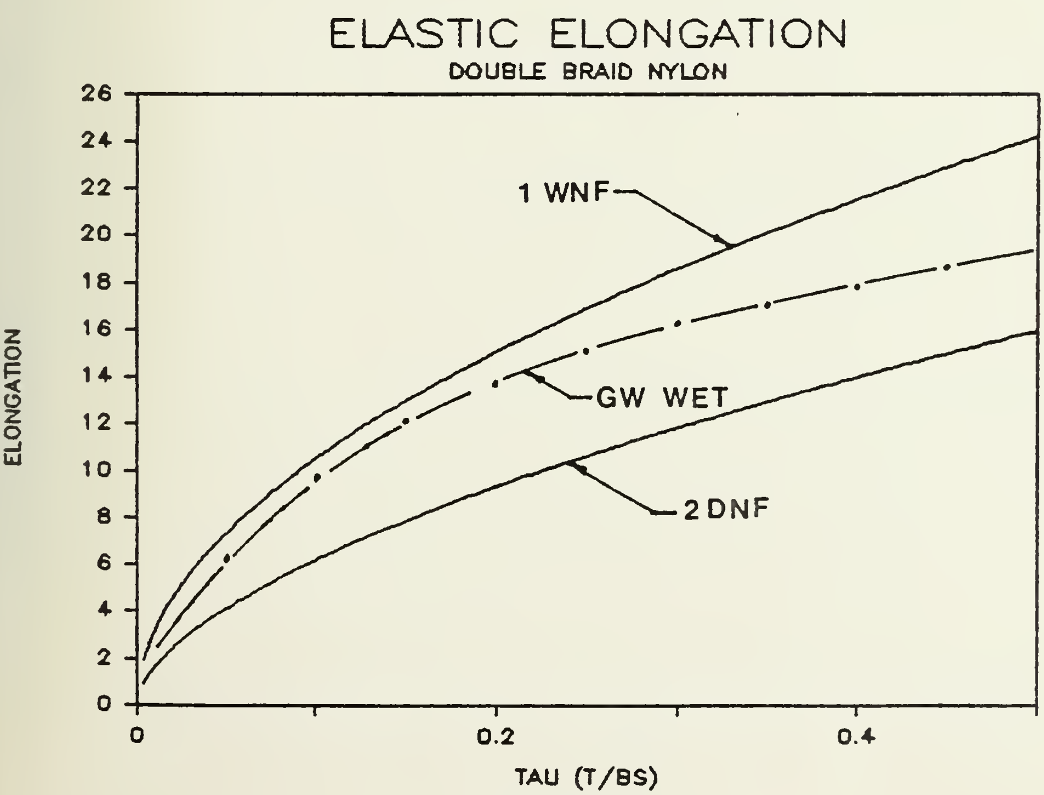

FIGURE 7

COMPARISON OF DESIGN MODELS WITH WET ROPE DATA REPORTED BY GIBSON AND WOLFE

describe their behavior [1]. These functions are based on the stabilized rope and caution should be used when applying them directly to very high cycle loading conditions because of the possible continued creep migration of the hysteresis loop. Test data on large polyester and polypropylene is not yet available to attempt to verify either function.

Chapter III

FACTOR OF SAFETY

3.1 Definition

Modeling elastic elongation as a function of specific loading which requires that the working load be approximated and that the breaking strength of the rope be known, implies that the size of the rope is known apriori. This is not the case, since the objective of the design is to size properly the rope used in making the towline. An alternate approach is to obtain a nominal specific loading by defining an acceptable factor of safety. Defining the ratio of the breaking strength to the working load as the factor of safety, FS:

$$FS = B_s/T_w \qquad (8)$$

which for nylon must be further modified to account for the reduction in breaking strength due to immersion in the marine environment. A range of 10% [3,4] to 20% [8,4] of breaking strength is commonly accepted with a value of 15% commonly used [1, 2, 6]. Using 15% here, the Marine Factor of Safety for nylon is defined as:

$$MFSN = 0.85 \times FS \qquad (9)$$

This implies that the specific loading will be:

$$\tau = 1/FS = 0.85/MFSN \qquad (10).$$

3.2 Range

Factor of safety values range from 1.5 [9], for single-point-mooring hawsers, to 9 [10] for general applications. For double braid nylon rope being used in high-cycle towing applications, there are several considerations which can be used to narrow this range.

It has been found that tensile fatigue failure (due to creep) is not so great a problem when ropes are cycled below 30% of breaking strength, if the minimum load is allowed to

go to zero during each cycle. It has also been shown that the rope can be cycled as high as 60% of breaking strength if the minimum load is never allowed to reach zero and will survive a comparable number of cycles. A load of 3% of breaking strength is cited [3,9] as an acceptable lower limit in this case. McKenna [1] recommends that the maximum load never exceed 30% of breaking strength for safety and that cyclic loads not exceed 20% of breaking strength.

There are also indications that under fast dynamic loading conditions, the rope will stiffen and have an apparent modulus several times that obtained from moderate cyclic load tests, magnification factors of 3 to 4 are cited by Bitting [3]. While McKenna [1] states, "Loading rates should not exceed 5% of the breaking strength per second with a delay of no less than ten seconds between cycles to approach a normal response", in towing applications, the delay between cycles will be dictated by sea state and heading, and it may not be possible to meet the ten-second recommended minimum, while a properly sized towline could remain below a load rate of 5% of breaking strength per second augmented by adjusting the speed of the towing vessel. In the event of the occurrence of a fast-dynamic load, the larger diameter rope, having a higher breaking strength and greater stiffness, would significantly increase the load transmitted to the other components of the system if a dynamic stiffness of 3 to 4 times were realized. Stiffness is also known to increase with extended exposure to water; this increase can be as high as 99% [3] with a concurrent strength loss of 50%.

The towing hawser will be a substantial investment and those used by the U. S. Coast Guard are expected to last for up to five years, during which time it is often exposed to water. The very nature of marine towing is one of cyclic loading and the possibility of minimum loads that approach zero cannot be excluded (such as a low speed tow in a high sea state). The combination of these constraints imply that loads should be kept below 30% of breaking strength, but that the minimum design load should be greater than 3% of breaking

strength range to limit the possibility of zero minimum cyclic load conditions. Thus, by examining the behavior of double braid nylon rope, a range for the factor of safety and the acceptable specific loading has been established:

$$3 <= FS <= 30 \quad (11)$$

$$3.5 <= MFSN <= 35 \quad (12)$$

$$3\% <= \tau <= 29\% \quad (13).$$

Note that the lower limit is maintained to account for the apparent effects of internal abrasion, caused by relative motion between the structural elements of the rope. Maintaining a minimum load has the effect of tightening the total structure and reducing the structural relaxation which results in the major component of relative motion.

3.3 Dynamic Load Amplification Factor

The upper limit, as stated, must include both the static and dynamic loads. In a quasi-static preliminary design the dynamic loads can be accounted for through the use of an assumed dynamic amplification factor. The magnitude of this dynamic amplification factor must be selected for each individual towing configuration based on an analysis of the relative accelerations between the towing and the towed vessels. For a towing system where the towed body is a submerged submarine, which is not excited by waves, the only source of dynamic loading is from the towing vessel. Under these conditions Kline and Blockwick [6] use a dynamic amplification factor of two which is based on towship motions obtained from scale model testing.

The range for τ can now be used in conjunction with the elastic-elongation function, curve (2) of Fig. 3, with predicted loads and displacements to size properly the required towline in diameter and length. This is done by modifying the upper limit by dividing by the appropriate dynamic amplification factor, which will be taken as two for the purpose of this presentation:

$$3\% <= \tau <= 15\% \quad (13a).$$

Chapter IV

QUASI-STATIC MODEL

4.1 Towing Configuration

A marine towing evolution in the presence of waves is obviously a dynamic environment and an exact evaluation of the loads imposed on the components of the towing system would include a combination of the static loads, due to the steady motion of the towed vessel through the water, and the dynamic loads of the time-varying wave excitation. Dynamic analysis of the towing system is an area of current research and is beyond the scope of this presentation. A quasi-static approach will be used here, with the application of the dynamic amplification factor imposed on the specific loading to account for wave excitation.

The model will be developed around an assumed towline profile for a submerged towing operation similar to that shown in Fig. 8. The towed vessel, at Point A, will be modeled as a notional submarine having dimensions as shown in Table 1.

TABLE 1

Design Point Geometry and Constraints

Dimensions		Design Constraints	
Length	200 ft.	Maximum Submerged Towing Velocity	15 knots
Beam	25 ft.	Design Tow Depth	200 ft.
		Maximum Vertical Tension T_{av}	15,000 lbs.
		Nominal Vertical Tension T_{an}	7,000 lbs.

Thus, the horizontal component of the towline tension at Point A, T_{ah}, can be calculated from the resistance equations given by Jackson [13]. For a specific design the desired depth and speed of tow would be dictated by operational requirements. In this notional case a maximum tow velocity of 15 knots with a design tow depth of 200 ft. will be assumed

TOWING PROFILE

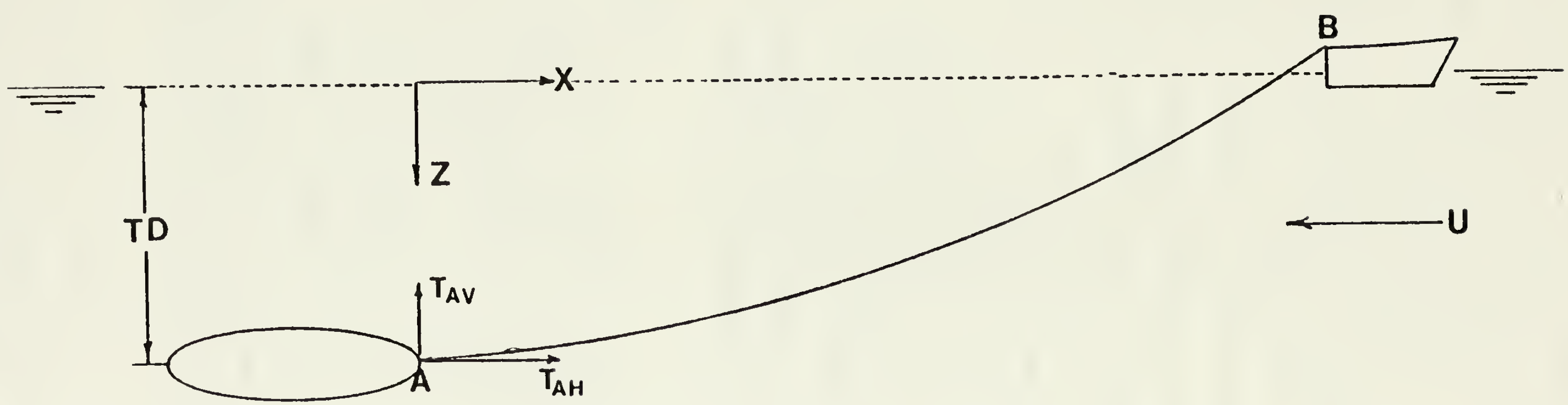

FIGURE 8

ASSUMED SUBMERGED TOWING SYSTEM CONFIGURATION

as a design point. Under actual towing conditions the vertical component of the towline tension must be compensated for by a countering force on the control surfaces. The range of possible control surface forces would depend upon the flow velocity, the size of the control surface, and the angle of deflection. Since these conditions would be specific to each case, a maximum vertical tension component, T_{av}, of 15,000 lbs will be assumed with an optimum value of 7,000 lbs during normal operations at the design point velocity of 15 knots.

For a given tow velocity, U, the actual velocity of the towed vessel and towline relative to the water, V, would be the algebraic sum of the towship velocity and the current velocity, v. It will be assumed that the local current velocity is small compared to the velocity of the towline through the water and that the relative velocity is equal to the towship velocity. It is recognized that this is a weak assumption for actual operations which could be conducted under conditions where the current velocity may be of the same order of magnitude as the towing velocity. However, if the correct relative velocity were known, it could be used in place of the towing velocity to improve the design.

4.2 Governing Equations

By removing an arbitrary element from the towline, as in Fig. 9, and establishing a local coordinate system which is everywhere normal and tangential to the towline axis, the governing equations will be:

$$dT/ds = (W-B) \times \mathrm{SIN}(\Phi) + F_t \qquad (14)$$

$$T \times d(\Phi)/ds = (W-B) \times \mathrm{COS}(\Phi) - F_n \qquad (15)$$

where T is the effective tension, as shown in Eq. (15a), which includes the hydrostatic pressure acting on the local cross-sectional area:

$$T = T_w + \rho \times g \times z \times A_w \qquad (15a)$$

The hydrostatic component is included here to offset the equivalent opposing pressure term in the elemental buoyancy.

TOWLINE ELEMENT

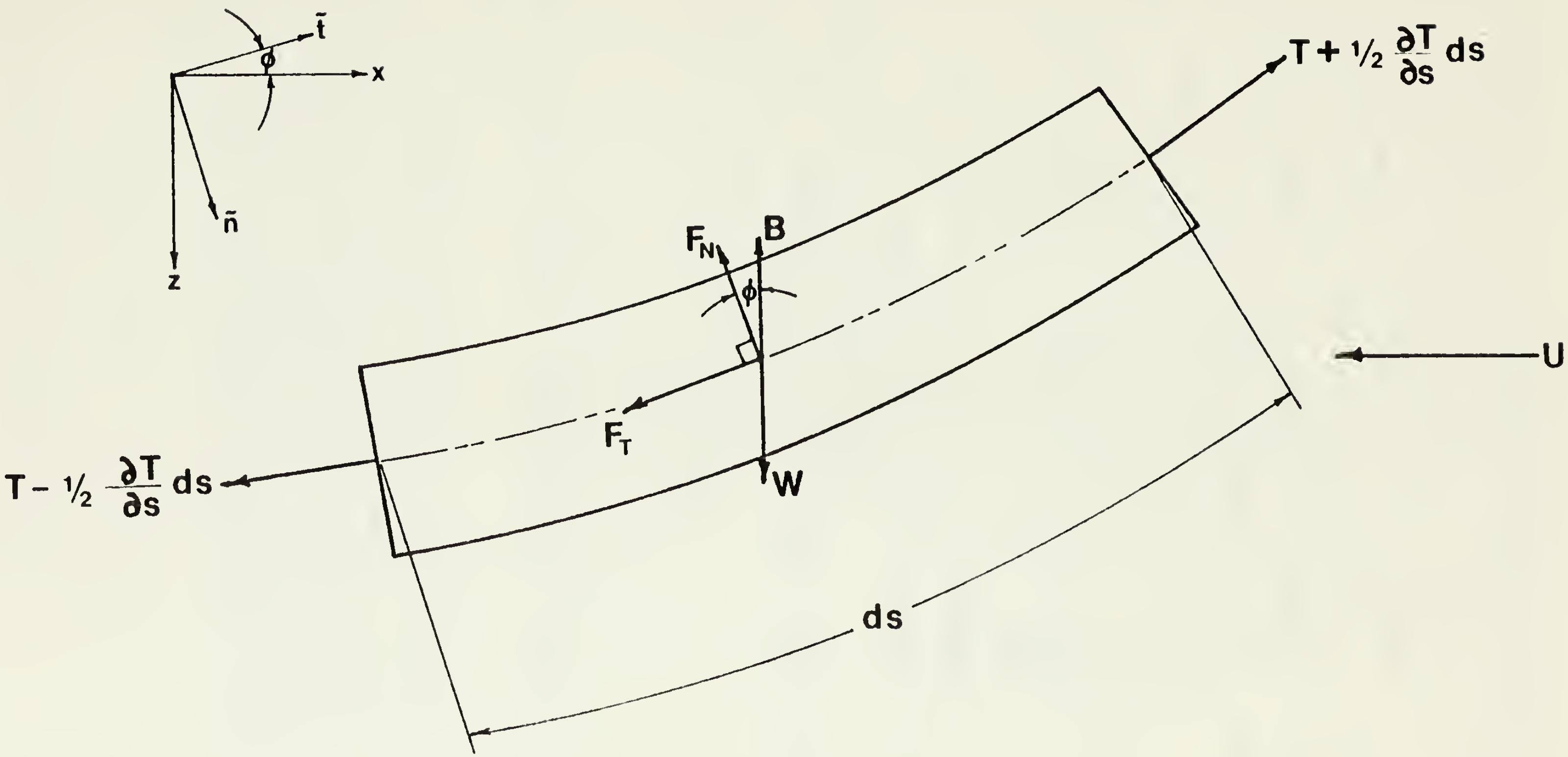

FIGURE 9

DIFFERENTIAL ELEMENT OF TOWLINE

The effective tension, T, is the appropriate tension to be used in elastic elongation calculations because it accounts for the Poisson effect of the hydrostatic pressure. However, it remains an approximation because it is applied to the total rope volume as if it were a linear elastic solid cylinder. The working tension, T_w, should be used in calculating the specific load in the cases where strength limitations are of greater concern than total elongation, or for cases where the depth is great enough to make the pressure contribution significant. For the rope sizes and the depths of interest in this preliminary design the hydrostatic component is considered to be negligible, i.e. on the order of 0.4% of B_s, and the effective tension will be accepted as the working tension. This results in a more correct elongation model and a conservative strength model.

F_n and F_t are, respectively, the normal and tangential hydrodynamic drag forces, defined as:

$$F_n = \tfrac{1}{2} \times \rho_w \times C_n \times d \times V_n^2 \qquad (16)$$

$$F_t = \tfrac{1}{2} \times \rho_w \times C_t \times PI \times d \times V_t^2 \qquad (17)$$

The equations are further simplified by defining the weight in water as:

$$W_w = (W-B) = P\tfrac{1}{4} \times d^2 \times g \times (\rho_\ell - \rho_w) \qquad (18)$$

The magnitude of the drag coefficients, C_n and C_t, is also an area of current synthetic line research and, again, approximations as found in Newman [14] and Springston [18] for the condition that flow-induced vibrations are not significant, are:

$$C_n = 1.0 \qquad (19)$$

$$C_t = (0.03 \text{ to } 0.05) \times C_n \qquad (20)$$

The mean values will be used in this preliminary design,

$$C_n = 1.0 \tag{19a}$$

$$C_t = 0.04 \tag{20a}$$

The use of thses values implies the assumption that flow-induced vibrations are not significant. The validity of this assumption will be examined after a preliminary design has been completed and the position of the towline relative to the flow has been approximated.

Where vortex shedding is present, and excites towline motions normal to free-stream velocity, the normal drag coefficient has been found to be as high as 3.0 [16]. This condition is usually found when the flow field is nearly uniform and normal to the axis of the test cable and, as such, would probably be minimal during towing operations. The frictional drag has not been measured during such flow conditions, and the effect of flow-induced vibrations on the frictional drag coefficient is not known.

A reference condition of near zero velocity with a minimal tension of T_0 will be assumed for an elemental length of ds_0 and diameter d_0. This diameter is equivalent to the new rope diameter and with its new-dry breaking strength, B_s, would be the criteria by which the rope would be ordered prior to incorporation into the towing system.

When immersed in water, the nylon will "shrink", causing the length to decrease. Recent measurements taken from active U.S. Coast Guard towlines showed an average shrinkage of approximately 5%. This will be the value used in the presentation where the new-dry length will be reduced by 5% to produce the wet reference length. Once installed into the

towing system and subjected to the cyclic loads, the permanent elongation, and thus the working length, of the line will be established. As previously stated, the permanent elongation for wet double braid nylon is approximately 13% at a load of 20% of B_s. Further, if it is assumed that the double braid structure does not contain a significant amount of trapped air when loaded to T_0, the assumption of incompressibility can be used in conjunction with the permanent elongation to provide the following relation between the reference length, ds_0, and the working length, ds_w:

$$ds_w = 1.13 \times ds_0 \tag{21}$$

$$A_w \times ds_w = A_0 \times ds_0 \tag{22}$$

$$d_w = d_0 \times (1/1.13)^{0.5} \tag{23}$$

The elemental working length, ds_w, will be used throughout this thesis as the basis for local elastic elongation. The resulting relation for elastic elongation is given by:

$$e = (ds - ds_w)/ds_w \tag{24}$$

$$ds = ds_0 \times 1.13 \times (1 + e) \tag{25}$$

A second application of incompressibility combined with an assumption of small elastic elongation, less than 15%, as cited in [1] and [10], leads to the final equations needed to express the local diameter, d, as a function of the original diameter, d_0:

$$A_w \times ds_w = A \times ds \tag{26}$$

$$PI/4 \times d_w^2 \times ds_w = PI/4 \times d^2 \times ds \tag{27}$$

$$d^2 = d_w^2 \times ds_w/ds \tag{28}$$

$$d = d_w/(1 + e)^{1/2} \tag{29}$$

where for small e:

$$(1 + e)^{1/2} = (1 + e/2) \tag{30}$$

$$d = d_w/(1 + e/2) \tag{31}$$

$$d = d_0 \times [(1/1.13)^{0.5}/(1 + e/2)] \qquad (32)$$

Substitution of these relations and the relative velocity, V, into the governing equations leads to a system of equations which must be satisfied by any valid model of the towline:

$$dT/ds = W_w \times SIN(\Phi) + F_t \qquad (33)$$

$$T \times d(\Phi)/ds = W_w \times COS(\Phi) - F_n \qquad (34)$$

$$F_n = \tfrac{1}{2} \times \rho_w \times C_n \times d_0 \times [(1/1.13)^{0.5}/(1 + e/2)] \times$$
$$\times V^2 SIN^2 (\phi) \times ds_0 \times 1.13 \times (1 + e/2) \qquad (35)$$

$$F_t = \tfrac{1}{2} \times \rho_w \times C_t \times PI \times d_0 \times [(1/1.13)^{0.5}/(1 + e/2)]$$
$$\times V^2 \times COS^2(\Phi) \times ds_0 \times 1.13 \times (1 + e) \qquad (36)$$

$$W_w = PI/4 \times d_0{}^2 \times [(1/1.13)^{0.5}/(1 + e/2)]^2 \times$$
$$\times g \times (\rho_\ell - \rho_w) \times ds_0 \times 1.13 \times (1 + e) \qquad (37)$$

4.3 Method of Solution

The solution of these equations will be achieved with an iterative numerical scheme using the drag at the towed body, the initial length of the towline, and the specified minimum specific tension as the boundary conditions for a viable solution. A range of feasible towline angles is assumed at Point A, Fig. 8. The resistance of the towed body is either supplied as an input, or calculated, and the initial tension, at point A, is then provided by the relation:

$$T = R_t/COS(\Phi) \qquad (38)$$

This tension is then multiplied by the minimum specific loading, obtained from the factor-of-safety analysis, to provide the desired breaking strength of the towline:

$$B_s = T \times \tau_{min} \qquad (39)$$

Using regression analysis from the double braid nylon rope specification [10], the required breaking strength can be related to a nominal diameter in the form:

$$d_0 = (B_S/34148.5)^{0.5258} \tag{40}$$

where B_S is in lbs. and d_0 is in inches. This is an approximate relation which has a maximum error of 7%. However, the diameter and the breaking strength that result from the preliminary design will not coincide with standard available sizes; in which case, the next higher standard size should be used, provided that both the diameter and breaking strength exceed the values obtained from the design model.

Rearranging Eq. (4) produces a relationship for the local elongation as a function of the local specific tension:

$$e = 0.2119 \times \tau^{0.5848} \tag{41}$$

For each assumed angle Eqs. (39) and (40) are solved for each element, ds, along the length of the line. The local diameter and elongation are then used as inputs for Eqs. (33) through (37). The change in depth as a function of elemental length and local angle is then calculated over the length of the towline by the geometric relation:

$$z(j) = z(j - 1) + ds_0 \times SIN(\Phi)_{(j-1)} \times 1.13 \times \times (1 + e) \tag{42}$$

The system of equations is iterated until z(j) is equal to the desired depth of tow. A new breaking strength and diameter is calculated each time, with the final values being those that satisfy the initial design constraints. A Fortran program which can be used for preliminary sizing is shown in Appendix D. The results of this program for the design point given in Table 1 are presented in Fig. 10.

The actual towline profile is shown in Fig. 10a. The profile is nearly flat due to the minimal difference between the densities of sea water and nylon, and due to the normal drag force which tends to lift the towline. The variation in tension along the length is shown in Fig. 10b, where it

can be seen that the total increase of 20,000 lbs. due to the hydrodynamic drag is of the same order of magnitude as the drag of the towed vessel. Figure 10c shows the local specific tension for the quasi-static model, the minimum specific tension of 10% was an input parameter, and the resulting maximum specific tension is approximately 16.5%. This is slightly greater than the desired 15% limit given in Eq. (13a), but still leaves approximately 12% of the breaking strength to account for dynamic loading and to remain within the limits of Eq. (13).

Figure 10d shows the elastic elongation which has a local maximum of approximately 7%. When added to the 13% permanent elongation, the total elongation of 20% is 4% less than the elongation at break cited in [10] and approximately 7% to 10% less than that cited by Flory [4] for cyclic load tests.

The reduction in diameter due to local elongation is shown in Fig. 10e, and the local angle with respect to the horizontal is shown in Fig. 10f.

A typical output from the design program is given in Appendix E. For the selected design point and the notional submarine, the recommendation for a 1200-ft towline is given in Table 3. Using the recommended size we conclude that the best standard size is a 10-inch circumference, double braid nylon rope with dimensions also shown in Table 2. Further analysis of the towing system will be based on this standard size.

TABLE 2

Recommended Towline Size

Recommended Size	Standard Size
Diameter 3.22"	Diameter 3.25"
B_s 320,000 lbs.	B_s 322,000 lbs.

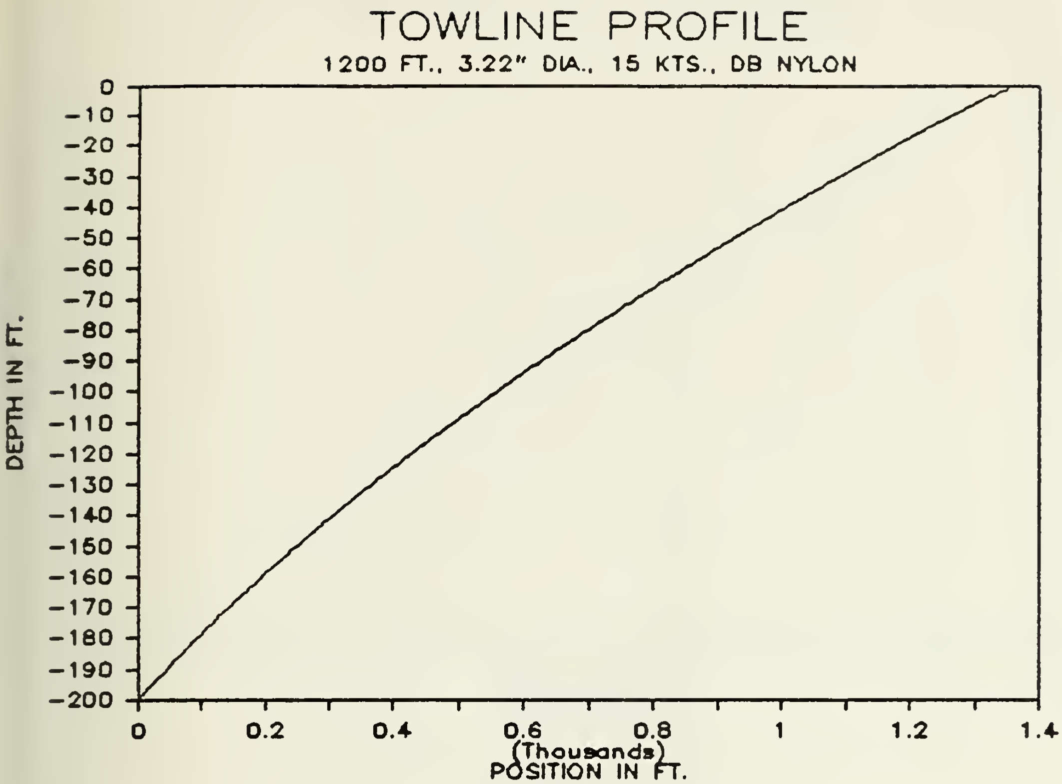

FIGURE 10a

TOWLINE PROFILE AT THE DESIGN POINT

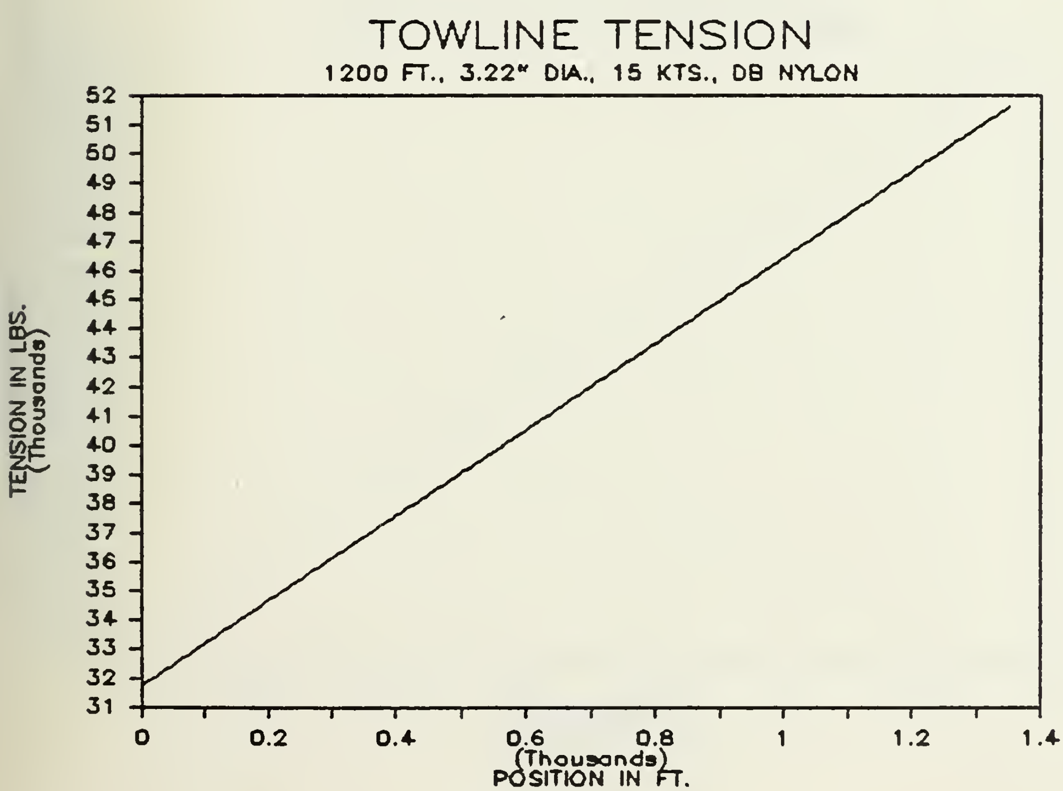

FIGURE 10b

TOWLINE TENSION AT THE DESIGN POINT

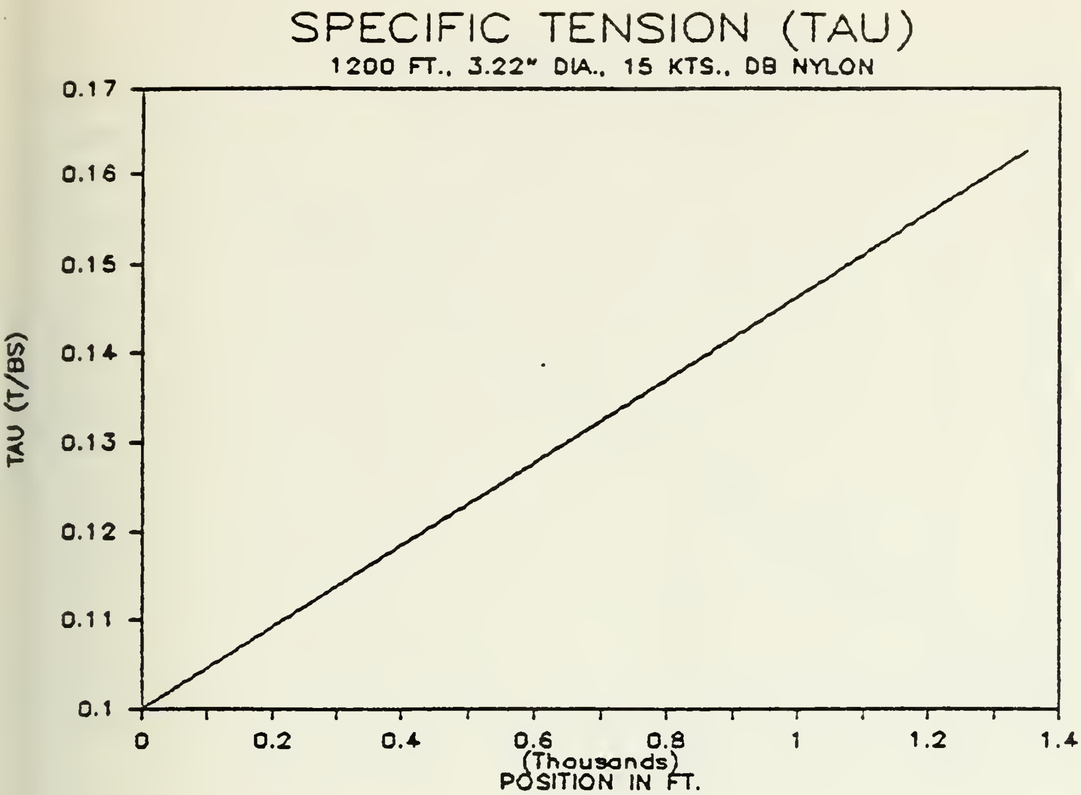

FIGURE 10c

SPECIFIC TENSION AT THE DESIGN POINT

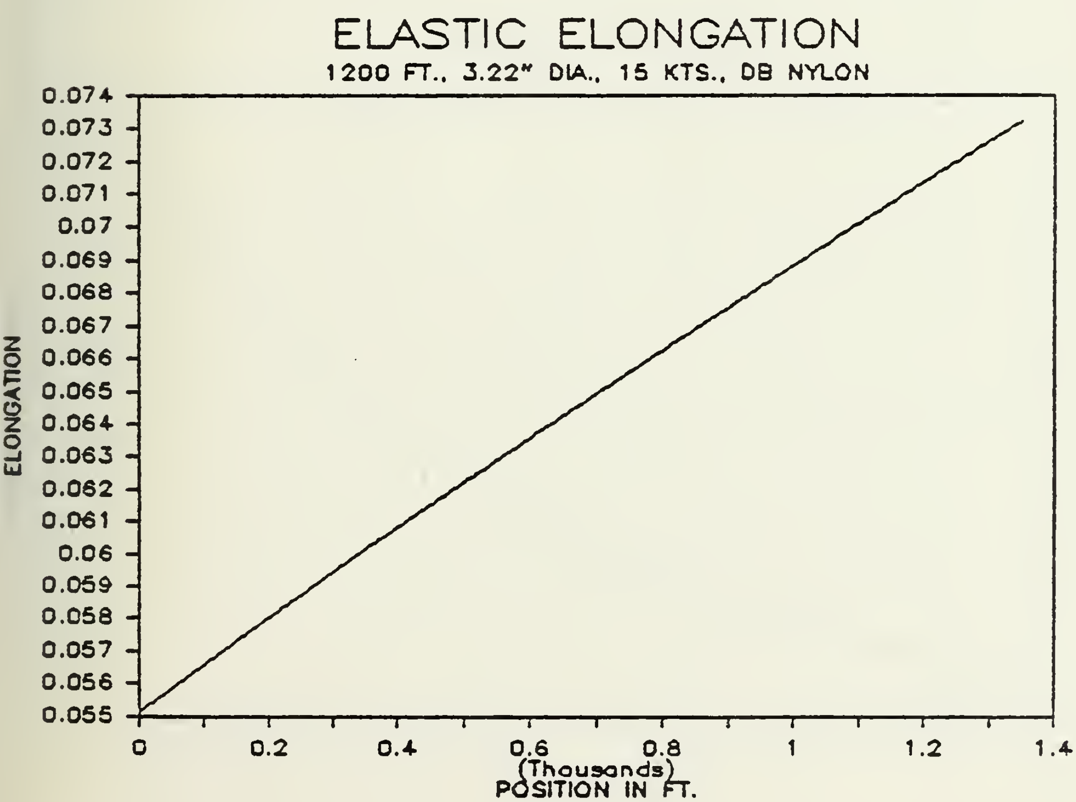

FIGURE 10d

ELASTIC ELONGATION AT THE DESIGN POINT

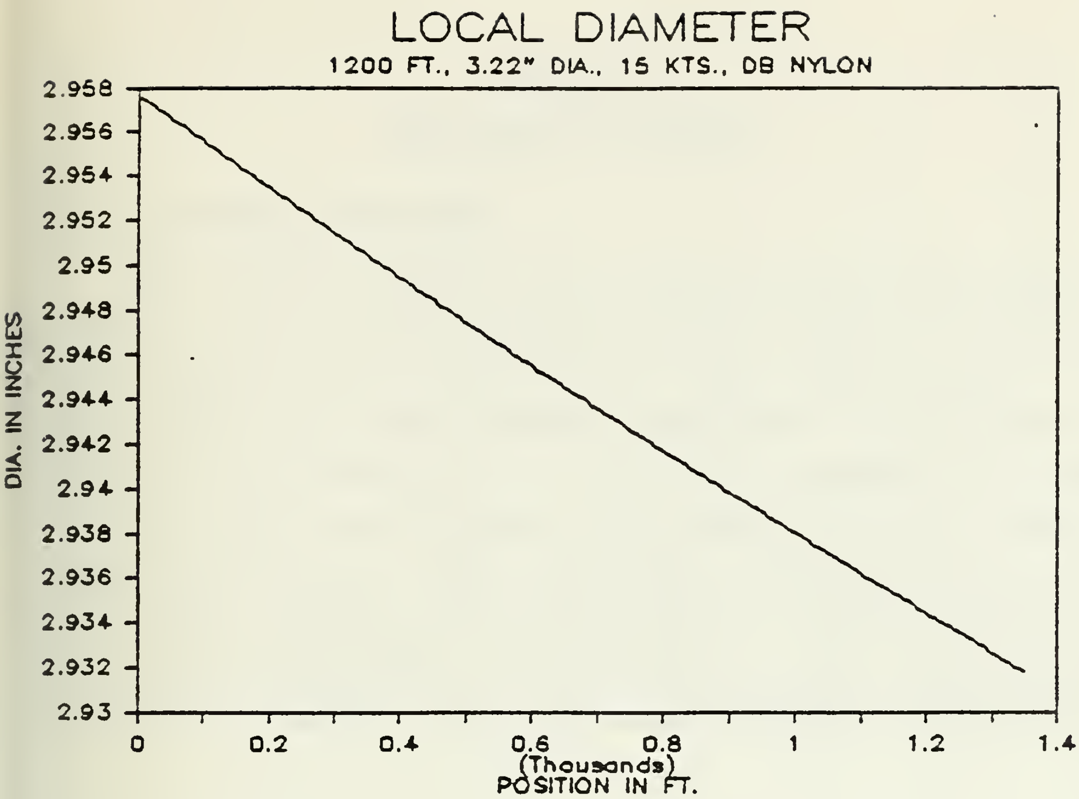

FIGURE 10e

VARIATION IN LOCAL DIAMETER AT THE DESIGN POINT

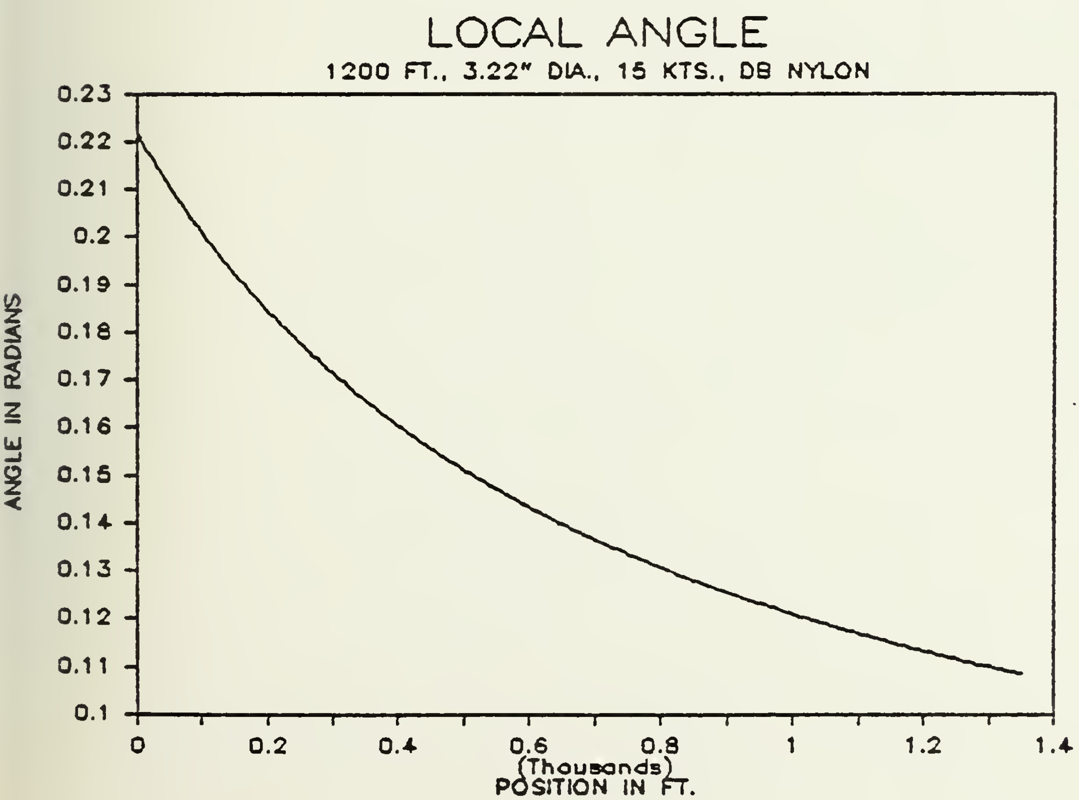

FIGURE 10f

LOCAL ANGLE AT THE DESIGN POINT

Chapter V

OFF-DESIGN ANALYSIS

5.1 Low-Load Operations

The initial selection of a design point has been the basis for calculating the preliminary size of the towline. These design point conditions are most often chosen to represent the worst case loading conditions and thus should result in the selection of a rope size capable of sustaining both the static and dynamic loads associated with extended towing operations. This engineering philosophy is valid for most of the commonly used materials in marine engineering and, indeed, provides a good starting point for polymeric materials. However, some polymeric materials (in this case, nylon), exhibit vastly different behavior when the loading regime is varied from that chosen as a design point. A complete preliminary design must also include an examination of off-design performance.

Some investigators [11] have recently found that the combination of relative movement between the structural elements of the rope and exposure to water can radically alter the abrasion characteristics of nylon filaments; the number of cycles to failure in laboratory experiments is an order of magnitude lower for wet-nylon yarns as compared to dry-nylon yarns. With comparable normal pressures and relative motion the same trend could be expected within the structure of the rope when it is immersed in water. This behavior has, in fact, been shown by Flory [4] in field experiments with eight-strand plaited nylon ropes of the size that could be used for towing vessels with a displacement less than 200 tons. These abrasion effects have also been reported by Bitting [17] in double-braid nylon ropes used as deep water buoy moorings. He notes that the ropes were subjected to low amplitude cyclic loads for extended periods, precisely the condition that the lower specific load limit is meant to avoid.

It is obviously impossible to preclude exposure to water in a marine towing system. Thus the only parameters that can be varied to limit the abrasion effects are the material, the local normal pressure, and the extent of relative movement. At some point in the future nylon may be replaced by polyester, or some other material, but this is at best several years away and may not happen at all, if other polyester characteristics, such as elasticity and temperature sensitivity, negate the possible gains in abrasion resistance. The remaining parameters (normal pressure and relative movement) can be somewhat controlled in the lower load regime by controlling the tension in the towline. However, these two parameters are not independent and cannot be considered separately. As the tension is reduced, the normal pressure between structural elements is also reduced and at some point the frictional forces will be too small to prevent the relaxation of the braided or twisted structure, thus allowing a greater degree of relative movement between the elements. During the next increasing load cycle the motion will be reversed and it is in this loading regime that abrasion resistance may have a pronounced effect. The lower specific load limit of 3% was imposed in the factor of safety analysis to account for this effect, but this limit only entered the design point analysis in an indirect manner by choosing a lower specific load limit of 10% in an attempt to stay above the 3% minimum during off-design operations. It is during slow speed submerged or surface towing operations that the lower limit becomes important.

In the submarine towing scenario used here the effects of varying the tow velocity are depicted in Figs. 11a through 11e. The initial tension in the towline at the connection point on the submarine is a function of velocity, which controls the hydrodynamic drag of the submarine, and depth of tow which dictates the initial angle of the towline. If, for the moment, we ignore the physical limitations of depth of tow and the vertical component of the towline tension, the depth of the submarine, in theory, can be

adjusted to create an angle which maintains a minimum τ of 3% in the towline which may inhibit the internal abrasion. The resistance of the notional submarine as a function of velocity is shown in Fig. 11a, and the tow depth that would maintain the required minimum specific tension is shown in Fig. 11b. The resulting initial towline angle is shown in Fig. 11c. As expected, the angle approaches a maximum of 90° as the velocity approaches zero; at this point, the depth of tow is equal to the stretched length of the towline. Figure 11d depicts the variation of τ along the towline, and it can be seen that the maximum value at the surface tow point is well below the limit given in the factor of safety analysis. However, at this point we must invoke reality and examine the vertical component of the towline tension which must be countered by the control surfaces of the submarine. Since the forces acting on the control surfaces are also a function of flow velocity, there will be a physical minimum speed and maximum depth of operation for each case. If, for the purpose of illustration, we assume that the control surface force is a V-squared function of the form:

$$F_{cs} = C \times V^2 \qquad (43)$$

For the notional submarine C is 67 and the control surface limit is reached between 7 and 8 knots, as shown in Fig. 11e. This translates into a depth limit of approximately 400 feet in Fig. 11b.

The depth of tow is further limited by the maximum operational depth of the specific submarine being towed; the designer would thus have to impose this limit of Fig. 11b. The resulting maximum depth, either the operational depth limit, or the control surface limit, would then reflect the minimum velocity that would maintain the required minimum specific loading.

The decrease in the abrasion resistance of nylon when exposed to water has been well documented. However, it should be emphasized that the exact magnitude of the lower limit for

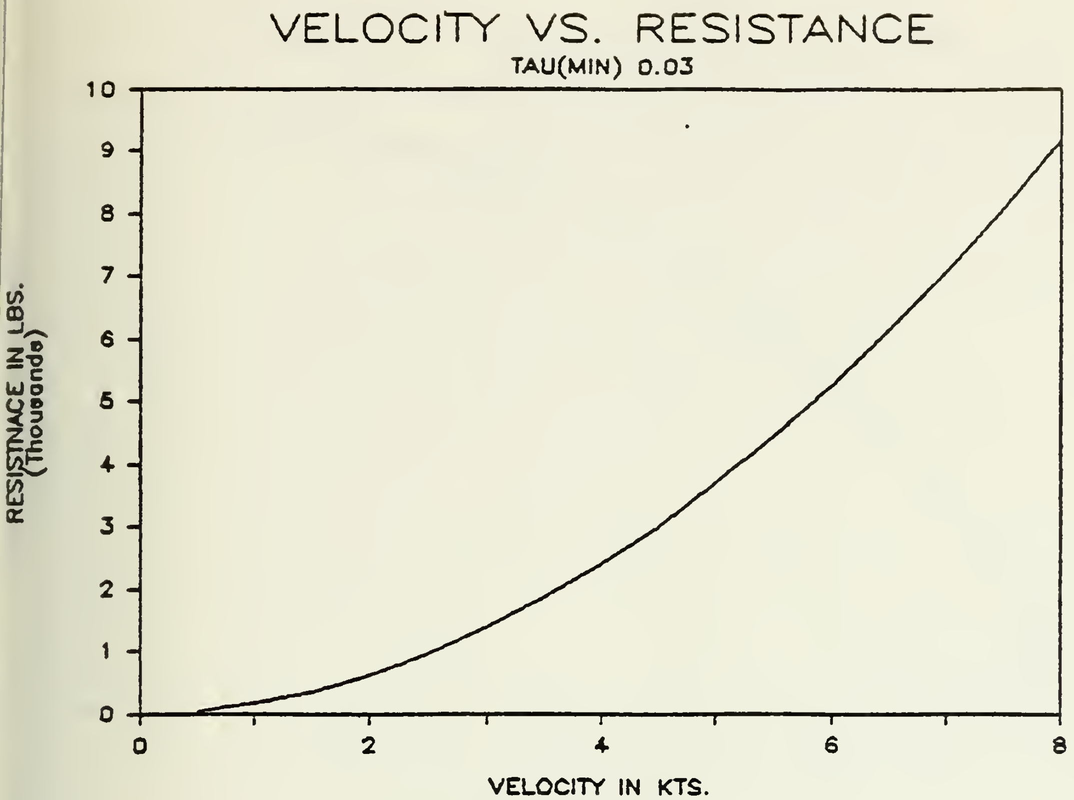

FIGURE 11a

RESISTANCE OF THE NOTIONAL SUBMARINE AS A FUNCTION OF VELOCITY

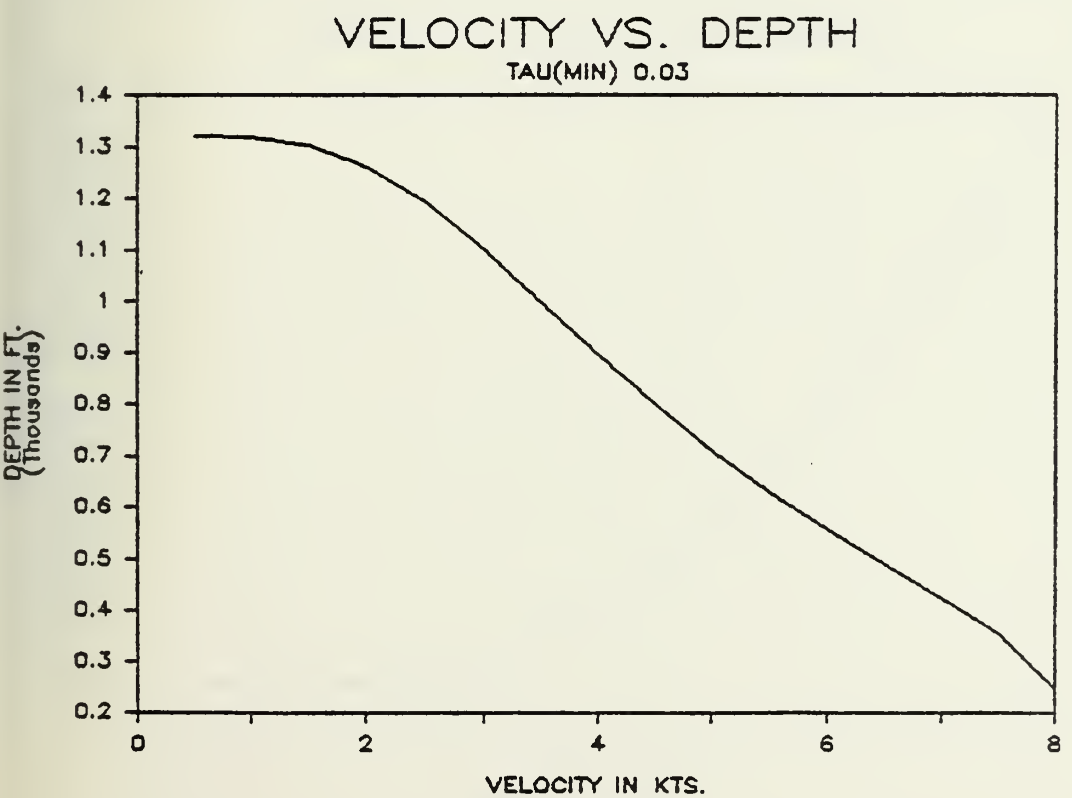

FIGURE 11b

DEPTH REQUIRED TO MAINTAIN 3% MINIMUM SPECIFIC TENSION AS A FUNCTION OF VELOCITY

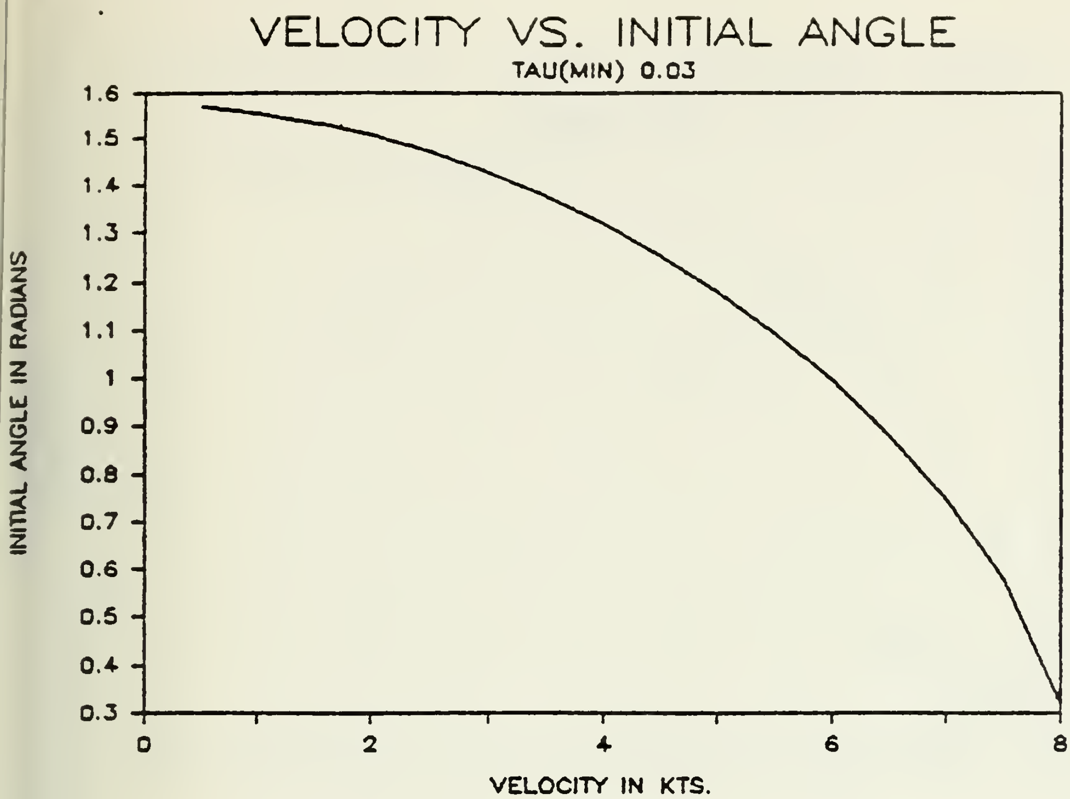

FIGURE 11c

INITIAL TOWLINE ANGLE AS A FUNCTION OF VELOCITY

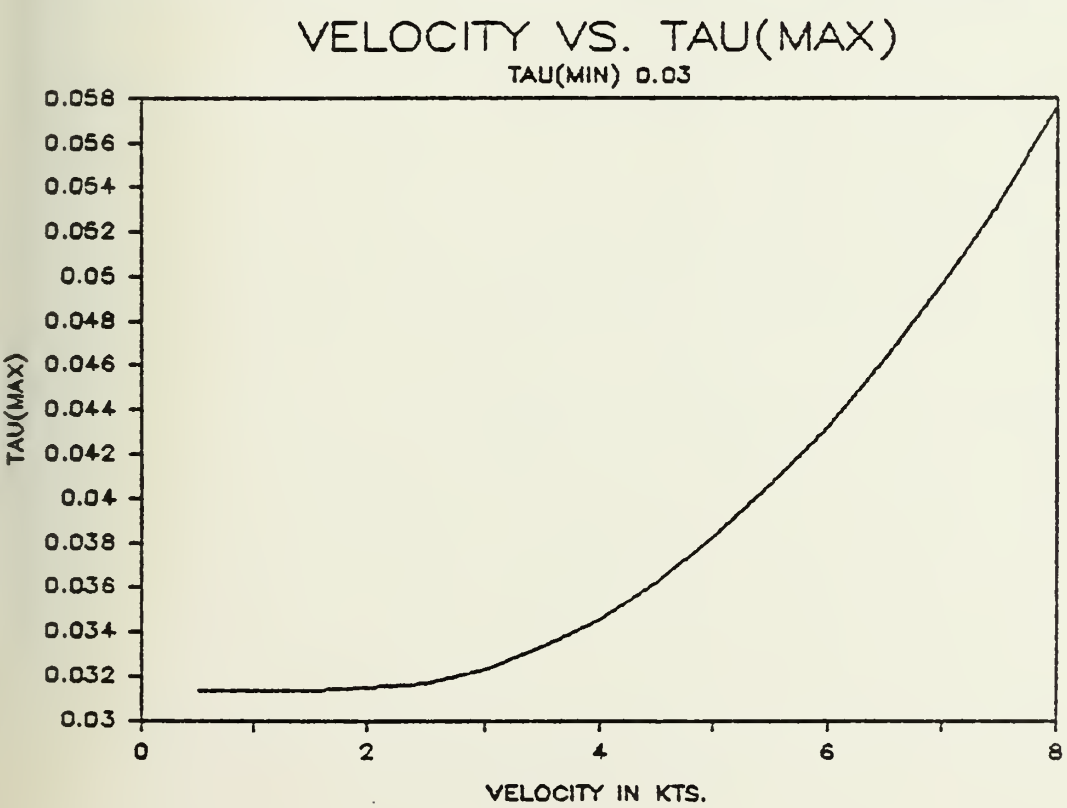

FIGURE 11d

MAXIMUM SPECIFIC TENSION AS A FUNCTION OF VELOCITY

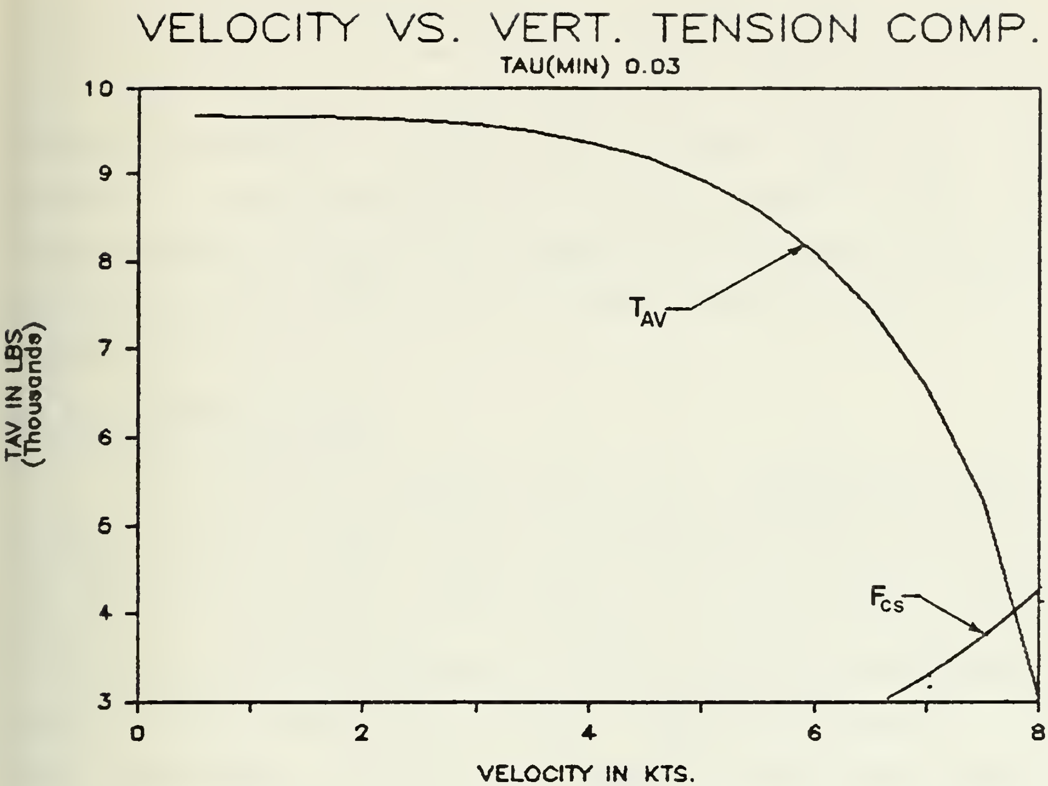

FIGURE 11e

VERTICAL TOWLINE TENSION AS A FUNCTION OF VELOCITY

a particular rope structure is not yet established. Continuing research into the wet abrasion characteristics of nylon may either raise or lower the limit in relation to the 3% of breaking strength cited by Bitting and used here. In general terms, it can be stated that any increase in the limit would only further restrict the operating envelope and any slight decrease would offer only minimal gains because the limit is already very low.

5.2 Surface Towing

It is highly unlikely that all submarine towing operations would be conducted with the submarine below the surface of the water. While entering and leaving port, or during emergency situations, the submarine would most probably be on the surface. Analysis of this scenario requires a knowledge of the positioning of the towing hardware on the bow of the submarine. If the tow point is above the water, the scope is short, and the towline is kept taut, the hydrodynamic drag of the towline would not be a factor, and the initial towline angle would be very nearly zero. In this case, the minimum tension is equal to the drag of the submarine and would be below the 3% minimum when the drag of the towed vessel is less than 3% of the breaking strength of the towline. For the notional submarine and the 3.25"-diameter towline, this would occur at a tow speed of approximately 7 knots. This is significant since all towing operations in restricted waters and during many emergency conditions would be conducted at less than 7 knots.

For the case where the tow point is below the surface of the water, a depth of five feet is assumed here, hydrodynamic drag on the towline must be accounted for. Since the towline is nearly parallel to the velocity vector, only the tangential drag component will be significant. Figures A1 through A4 in Appendix A, present the plots for towline profiles, towline angle, specific tensions, and elastic elongations at various towing velocities for surface towing with a submerged tow point. At 3 knots, Fig. A3 shows that the specific

tension is everywhere less than 3% and Fig. A4 shows that the elastic elongation is less than 2%. This is precisely the condition that would exist in most cases for entering and leaving port and it is a condition where abrasion could be significant. The same characteristics are shown for a tow velocity of 6 knots. The initial τ of 2.8% is just below the recommended minimum and depicts an acceptable towing condition. At 9 knots τ has increased to an average value of 10% with an associated average elastic elongation of 5.7%; both of these values are acceptable, but they are now approaching the maximum limit for total elongation, 24%, where the elastic elongation at the towing vessel is added to the assumed permanent elongation of 13%. The 12-knot curve is above the upper limit for τ (Eq. (13a)), and represents the maximum surface towing velocity for calm water where dynamic loads are not significant.

TABLE 3

Notional Surface Resistances

Velocity in knots	Resistance in pounds
3	4,000
6	9,000
9	30,000
12	60,000
15	75,000

The surface resistance characteristics given in Table 3 have been assumed for a notional submarine; an actual towing analysis would be based on known resistance data, or scale model tests. The notional submarine used here simply illustrates the possibility that a towline sized for a submerged velocity of 15 knots may be more severely limited for surface towing conditions. Here, we have an approximate surface maximum of 10 knots which could decrease even further if the assumed dynamic amplification factor was not large enough to account for wave excitation of both the towing and towed vessels.

5.3 Flow-Induced Oscillations

Flow-induced oscillations of long cylinders in a fluid medium is a dynamic phenomenon which has been investigated by, among others, Sarpkaya [15], for a stationary bluff body, and by Vandiver [16] for long flexible cylinders. Consideration in a quasi-static analysis is only to the extent that such vibrations have been reported to increase the normal drag coefficient by as much as 200% from 1.0 to a maximum of 3.0. Such increases have been measured in laboratory and field experiments where the flow was normal to the axis of the cylinder and under the following conditions:

i. $R_n <= 10^5$

ii. Spatially uniform cylinders

iii. Spatially uniform flows

iv. Low modal density.

Under these conditions it is possible to approximate the normal drag coefficient with some degree of confidence by using an empirical relation proposed by Griffin and modified by Vandiver in the form:

$$C_n = C_{no} \times [1+1.043(2Y_{rms}/d)^{0.65}] \quad (44)$$

where C_{no} is the rigid cylinder normal drag coefficient that has been used to this point and Y_{rms} is the RMS amplitude of cable vibration. If, for a worst case, we assume that the towline can be approximated by the above conditions and that the amplitude of vibration is $0.7 \times \Delta$, similar to that reported for the pipe by Vandiver [16], we find that the C_n maximum is 2.8. A normal drag coefficient of this magnitude would obviously result in a substantial increase in the size of the towline which would be capable of sustaining the maximum loads at the end connected to the towing vessel. Since the resistance of the towed vessel would remain the same for a given speed, it would also make it much harder to remain above the lower load limit at that end of the towline.

The Reynolds number, R_n, for the actual towline is shown in Fig. 12a. It has been calculated, based on the local

normal velocity, Fig. 12b and the local diameter, Fig. 12c. From these figures we see that the first and second conditions, $R_n <= 10^5$ and a spatially uniform cylinder, have met, but that the third condition, spatially uniform flow, is not met. Since the flow field is not uniform, we expect a very short vortex correlation length with a random vibration response [16]. Under these conditions, the amplitude of vibration and the associated drag are substantially reduced. C_n approaches C_{no} and a magnitude of 1.0 to 1.2 is found to be a good approximation for sizing the towline.

Figures B1 through B6, showing the effects on the required size and the equilibrium condition due to increasing the normal drag coefficient from that assumed for design, i.e. to a maximum of 2.5, are presented in Appendix B for the design point of 15 knots and 200 feet. In practice one would probably purchase the standard commercial size above that recommended by the design. A design based on a C_n of 1.2 would result in a 10-inch circumference towline, as was already recommended using a C_n of 1.0. In contrast, a design based on a C_n of 2.5 would result in the selection of a 12-inch circumference towline. This variation is not minor, since the increase in B_s is of the order of 40% and it would become impossible to keep the specific load above the lower limit for slow speed tows or during any configuration where flow-induced vibrations decreased or stopped and the normal drag coefficient decreased to approximately 1.0.

For all cases, the tangential drag coefficient was assumed to be constant at 0.04.

5.4 Dry- Versus Wet-Nylon Function

Earlier we examined the elastic elongation characteristics of double braid nylon and reached the conclusion that under high cycle load conditions the DNF better represented the data presented in the literature. It was also stated that the WNF would be more appropriate for a new line being placed in a towing system, and that it would provide an upper

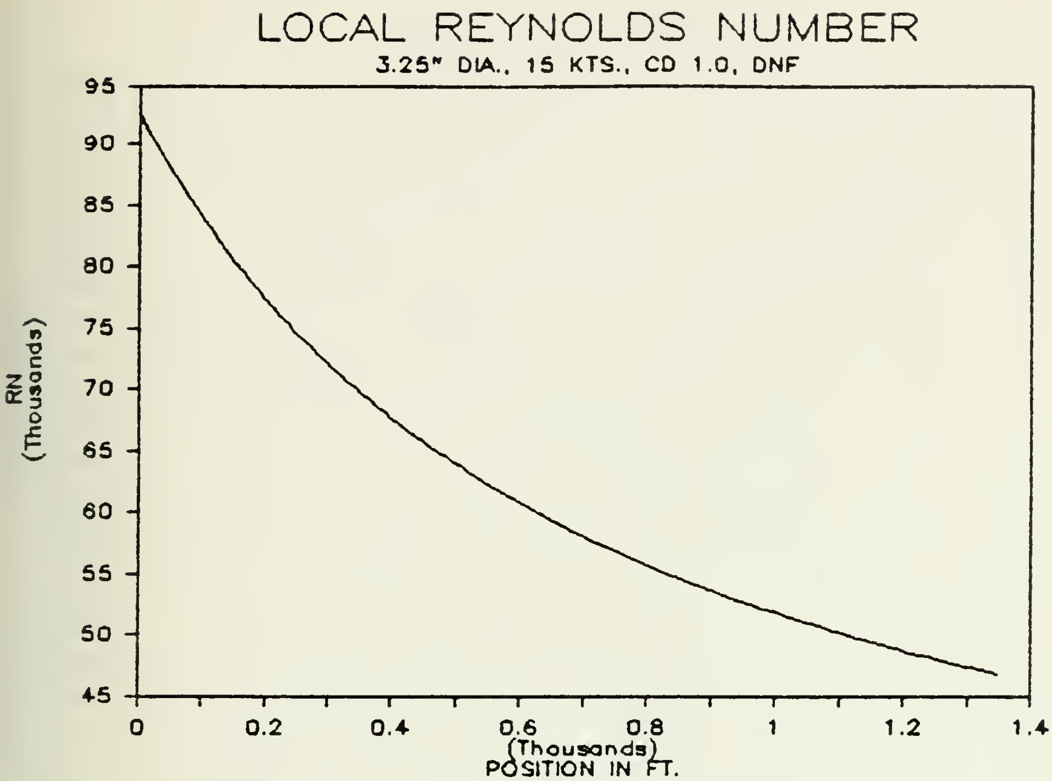

FIGURE 12a
LOCAL REYNOLDS NUMBER ALONG THE TOWLINE AT THE DESIGN POINT

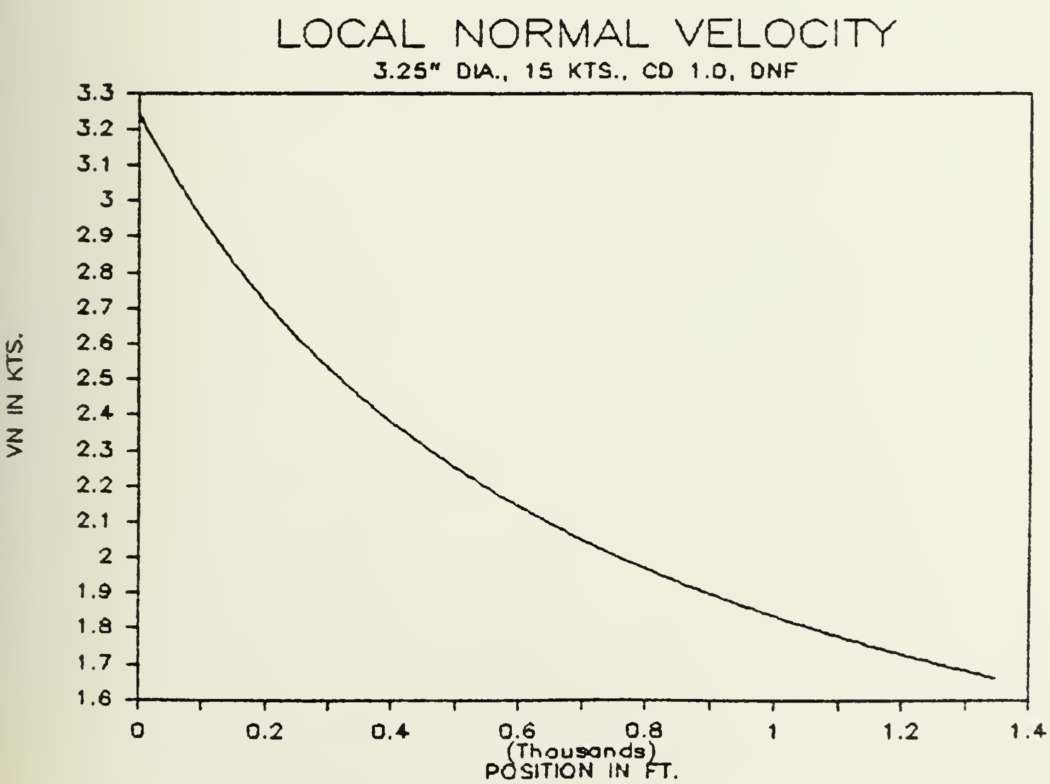

FIGURE 12b
LOCAL NORMAL VELOCITY ALONG THE TOWLINE AT THE DESIGN POINT

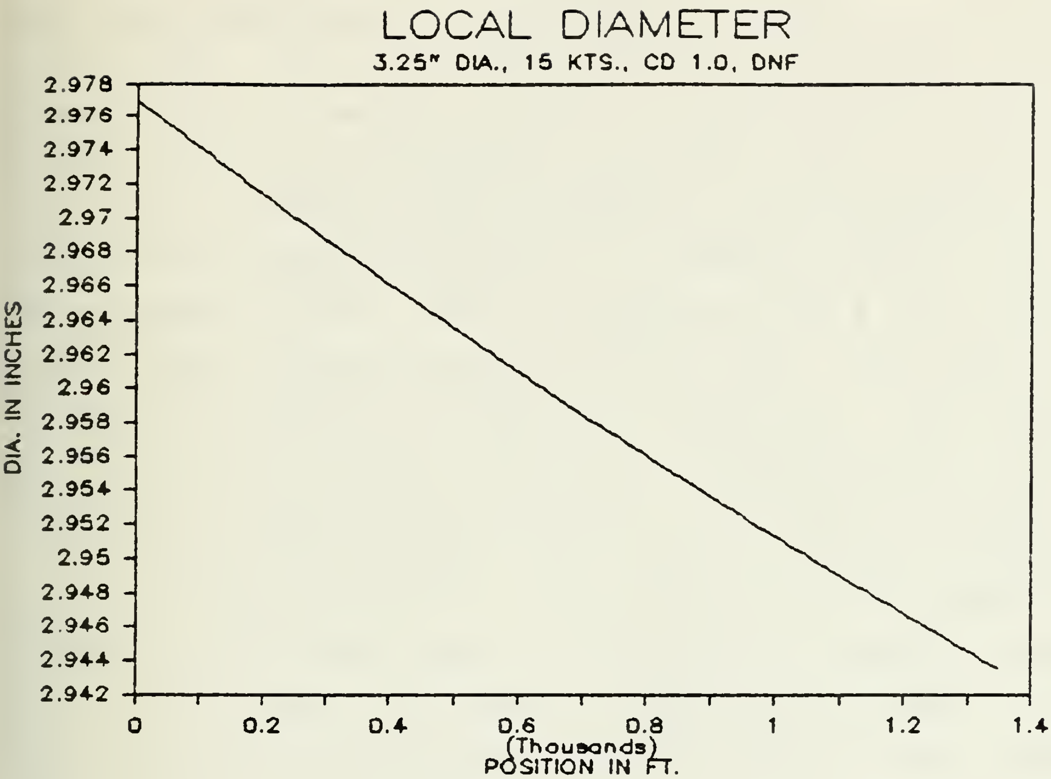

FIGURE 12c

LOCAL DIAMETER ALONG THE TOWLINE AT THE DESIGN POINT

limit on in-service elongation. However, the point at which the design switches from the WNF to the DNF has not been established. From Figs. 2b and 3 it would appear that this change could be made as early as the first few hundred cycles, but cyclic elongation data for double braid nylon rope used in marine towing systems is very limited, and a conservative approach would be to use the WNF for the first few thousand cycles. This translates into 4 to 8 hours of towing under normal operating conditions. The implications of using the WNF are presented in Appendix C, for operations at or near the 15-knot and 200-foot design point, and for slow speed, deeper operations at 3 knots and 600 feet.

As might be expected, the towline profile, towline tension, and specific tension are practically the same for both the DNF and the WNF, Figs. C1 through C3, and C11 through C13. In contrast, the elastic elongation in Figs. C4 and C14 show a significant increase when the WNF is used. Because of the increase in elongation there is a concurrent decrease in local diameter as shown in Figs. C5 and C15. At the design point, the maximum elastic elongation is approximately 13%, which, when combined with the assumed permanent elongation of 13%, implies a total elongation of 26%. This is an unacceptable value which leaves little or no elongation margin for dynamic loads. It indicates that a new towline has an operating point substantially below that of a used, but undamaged, towline.

The remaining figures show only minor effects from the increased elongation, and it can be concluded that upper range of the operating envelope is limited by local elongation rather than strength, as reflected in the specific loading and thus the factor of safety. However, it can also be concluded from these figures that a towline size based on the WNF, which would be substantially larger, would be very prone to the effects of internal abrasion, since it would continually operate at much smaller specific loads. The specific loading conditions shown in Fig. C13 would be typical of such operations even at higher towing speeds.

Chapter VI

CONCLUSIONS

6.1 Controlling Material Characteristic

In the preceding discussion it has become apparent that recent research [11] which has revealed the susceptibility of wet nylon to abrasion and the apparent greater dominance of this characteristic under low load conditions [17] is the most severe constraint on designing a nylon towline to operate over a wide range of loading conditions in the marine environment. It is theoretically possible to increase the size of a towline to account for larger static or dynamic loads, or to account for a greater degree of uncertainty, but, because of internal abrasion that has been observed in nylon lines, such a design approach simply changes the mechanism by which the towline fails and does not decrease the likelihood of failure.

6.2 Controlling Structural Characteristic

The controlling structural characteristic in this design model is the assumed form of the elastic elongation function. The DNF and the WNF result in significantly different elongation behavior. Because of established abrasion characteristics of wet nylon, neither function can be used to produce a conservative design over the total projected loading regime of a marine towline. Both of these functions are based on a minimal number of cycles and even though they appear to bound cyclic load data presented by other authors (for what are assumed to be wet testing conditions), it is noted that there are no truly wet tests where the rope has been continuously immersed in water, available for comparison. As a result, the applicability of the elastic elongation function to very high cycle wet loading conditions is questionable. Future research may provide better models which result in a much less constrained design. Until this research has been completed, some form of the elongation

behavior presented here must be used in conjunction with an experienced seaman's knowledge of what has worked well in the past.

6.3 Controlling Loading Parameter

A review of Figs. C2 and C10 shows that the appreciable increase in tension along the towline is primarily due to the frictional (tangential) drag of the towline. The magnitude of F_t is a reflection of the tangential drag coefficient, C_t. The value of C_t used here was suggested by Triantafyllou and supported by Springston [15], and is probably very close to the actual value, but it appears that there are very little data specifically for the frictional characteristics of synthetic ropes. It would require only relatively small variations of C_t to produce a fairly significant change in the maximum local loading and the associated maximum local elongation. This change would be beneficial if C_t were found to be less than that used here and would be detrimental if it were found to be greater. In either case, it would substantially improve the confidence in the final selection of the rope size.

6.4 Summary

One possible method of selecting the size of a double braid nylon rope for a marine towline has been presented here. This method is primarily based on behavior characteristics presented in the literature. It was not possible to reach a definitive conclusion because the model was eventually controlled by assumed values for the minimum acceptable loading condition and the appropriate hydrodynamic frictional drag coefficient. These assumed values have a basis in the literature, but further research is needed to establish their exact magnitudes for synthetic ropes of various materials and/or constructions.

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FIGURE A1

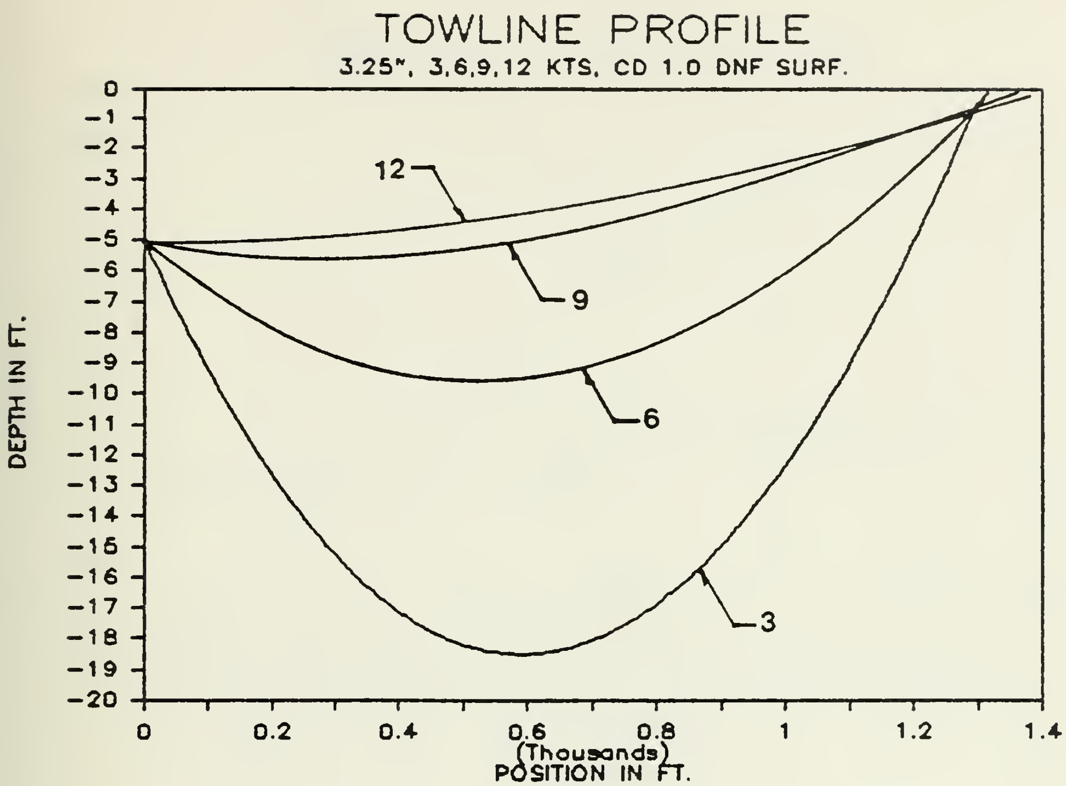


FIGURE A2

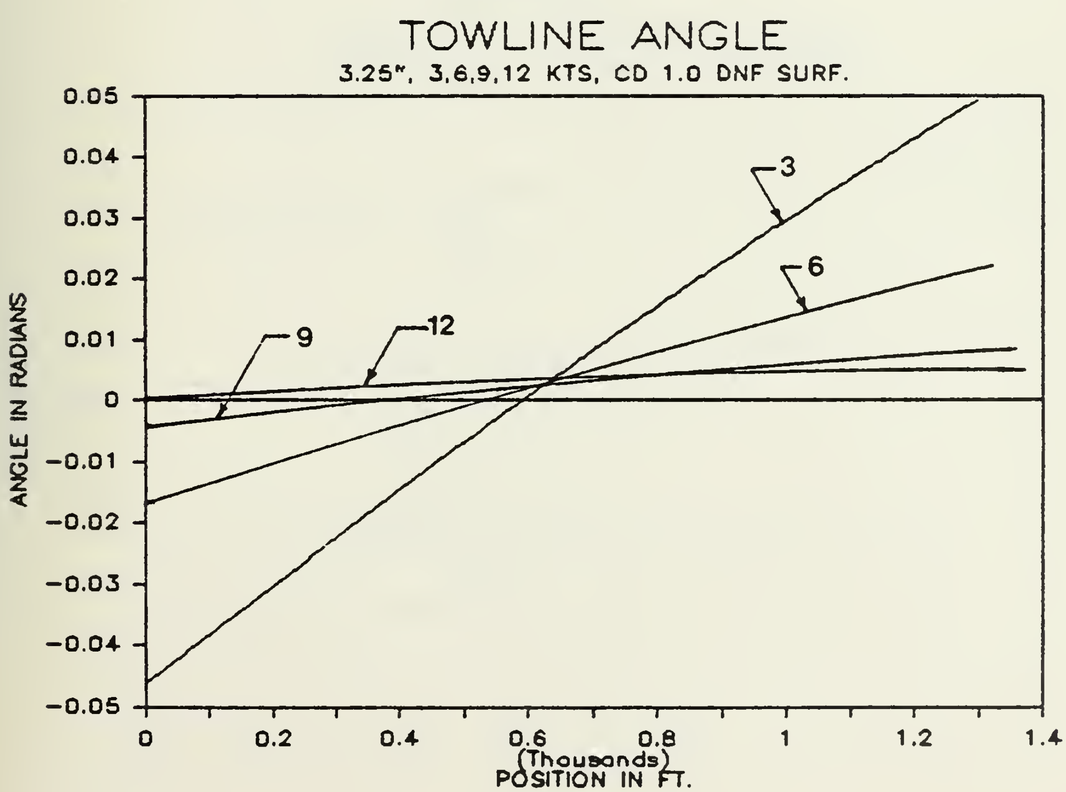


FIGURE A3

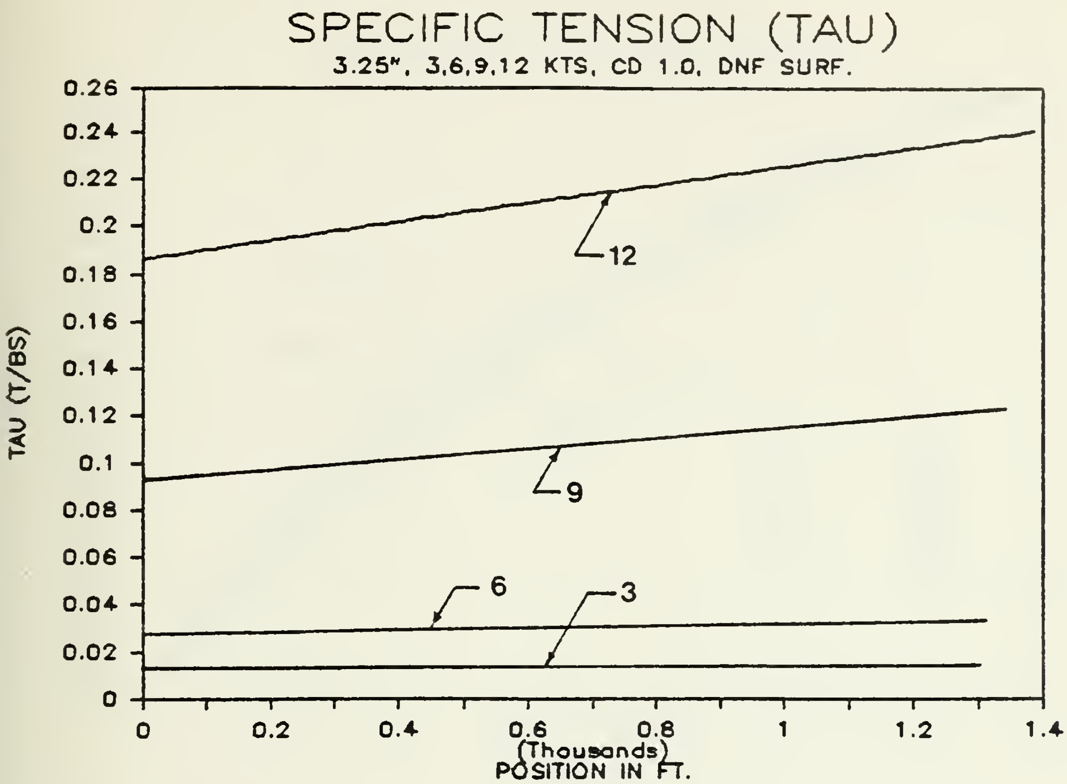


FIGURE A4

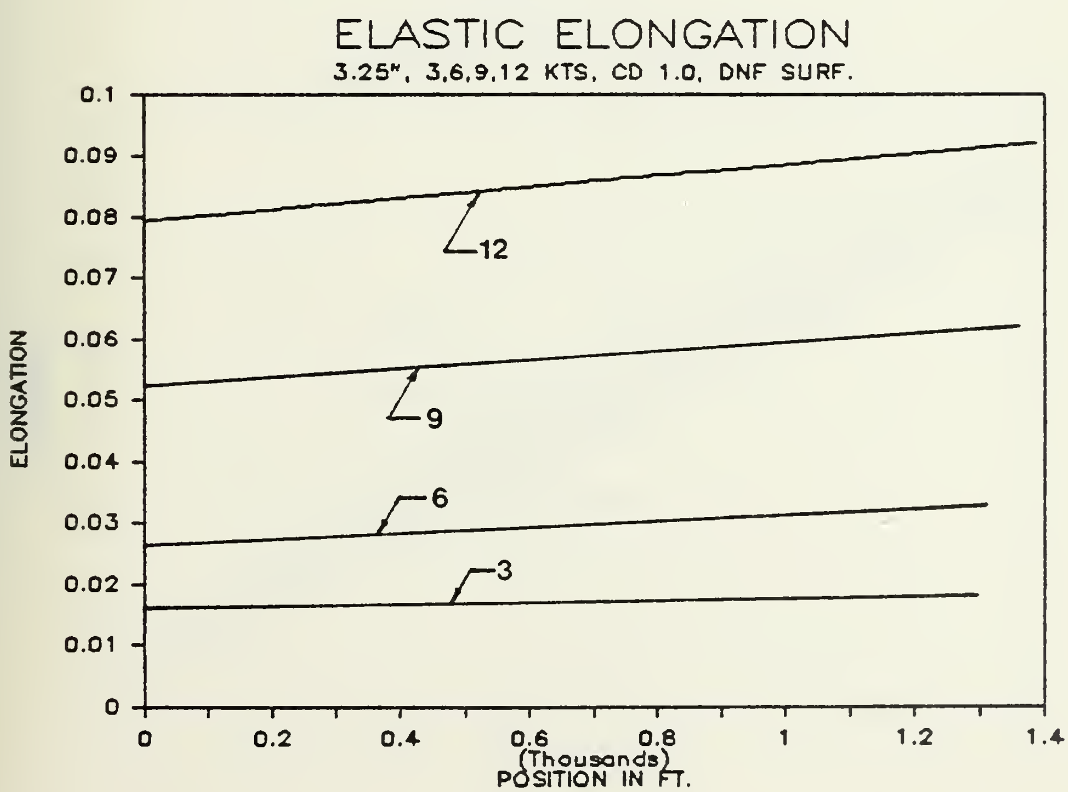


FIGURE B1

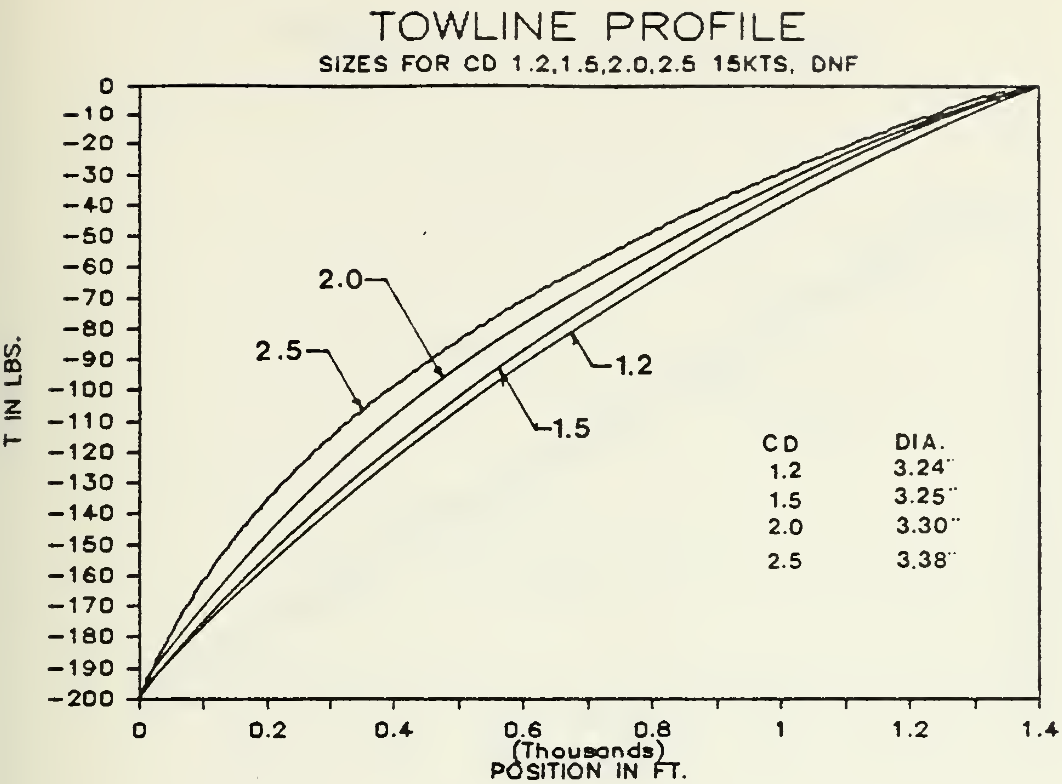


FIGURE B2

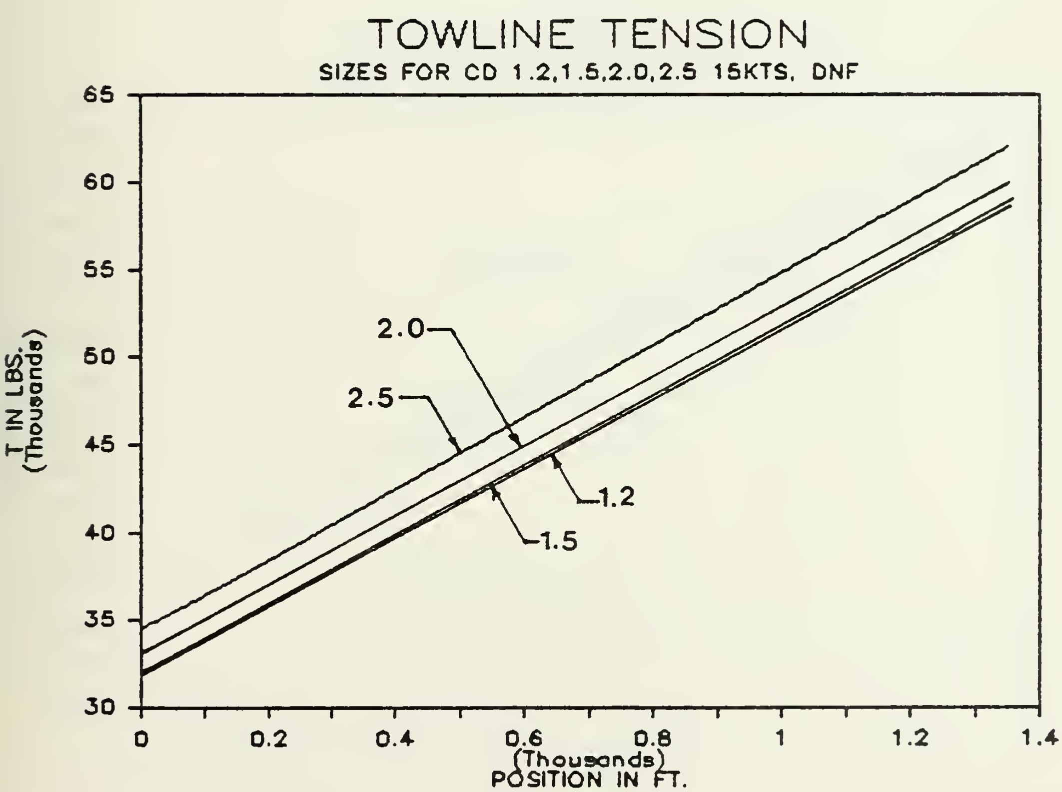
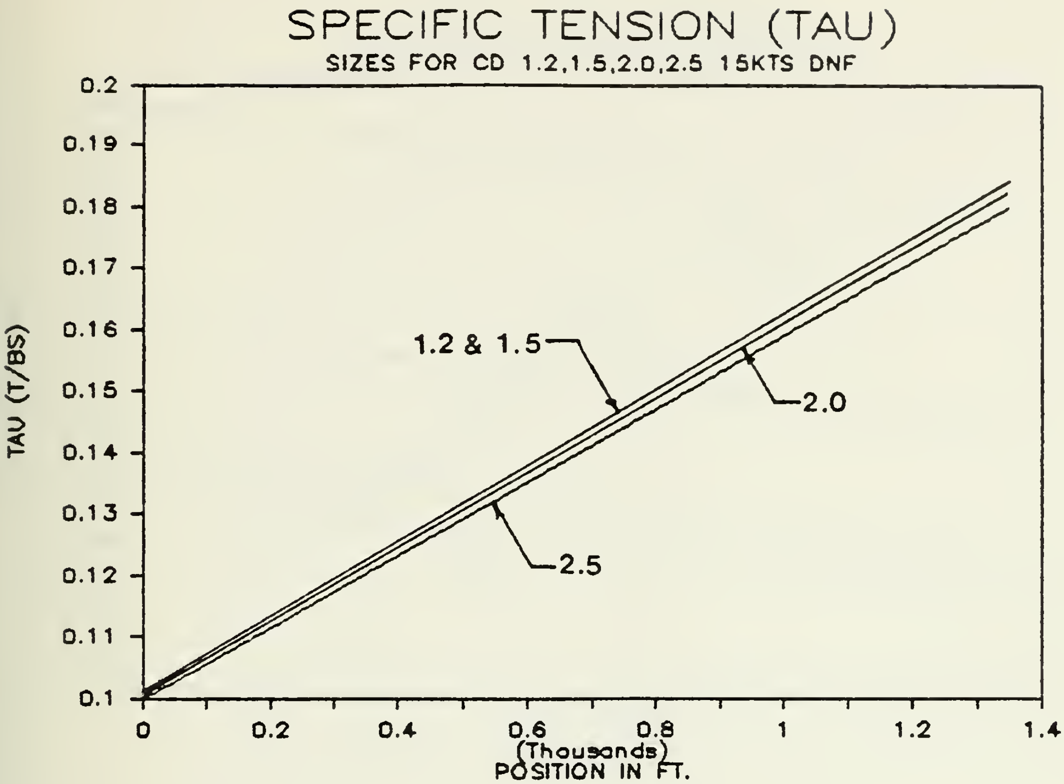


FIGURE B3



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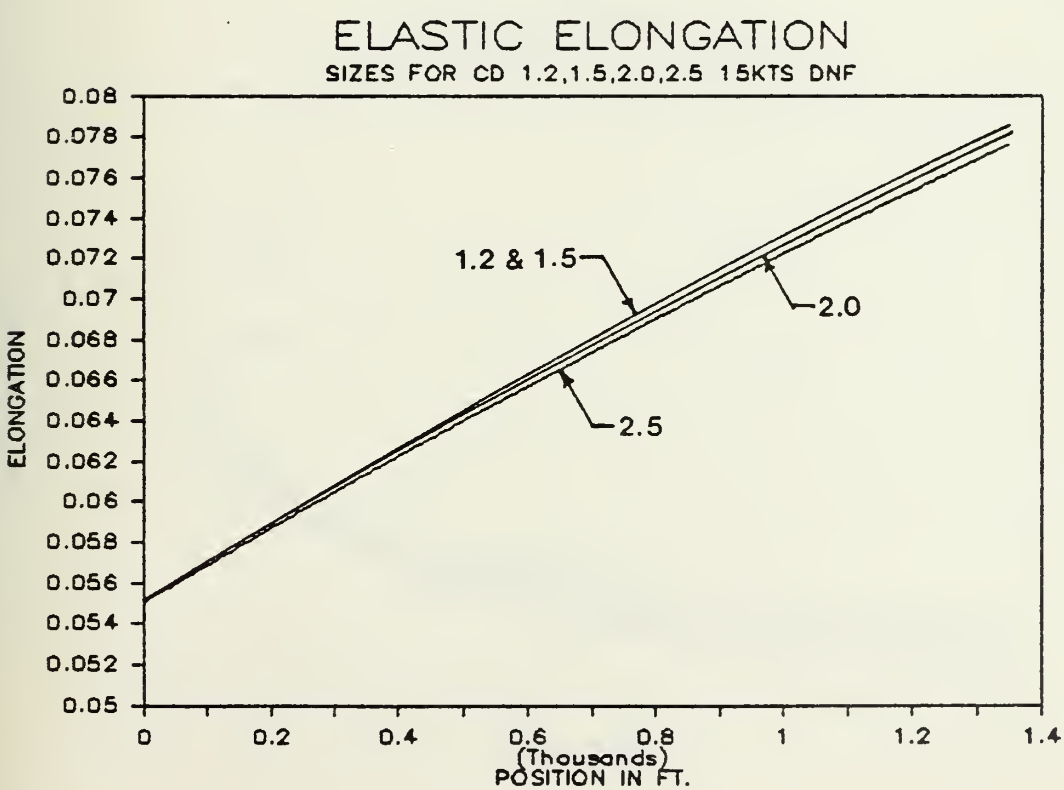


FIGURE B5

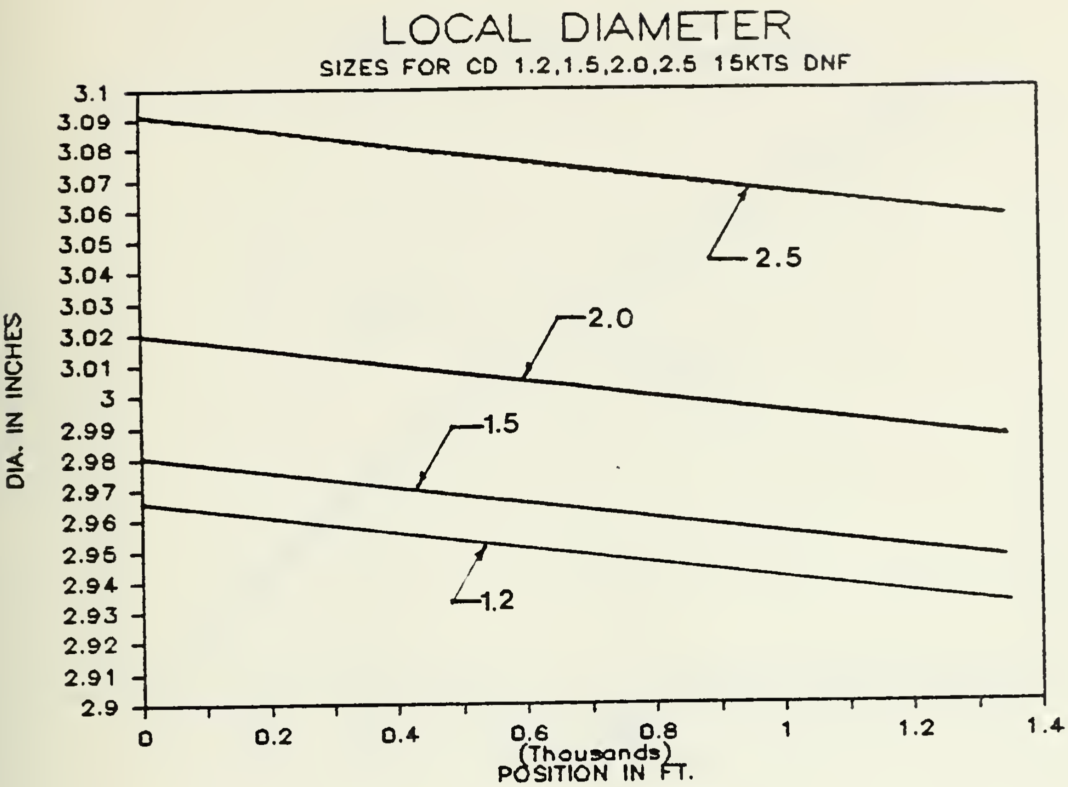


FIGURE B6

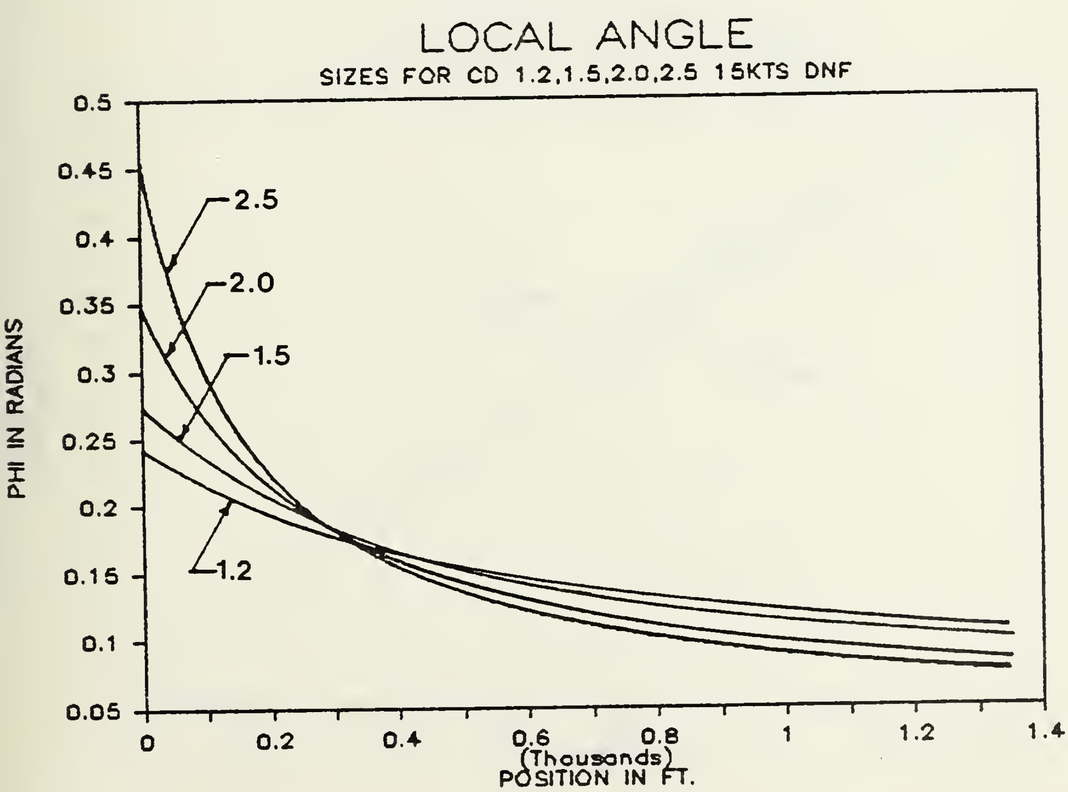


FIGURE C1

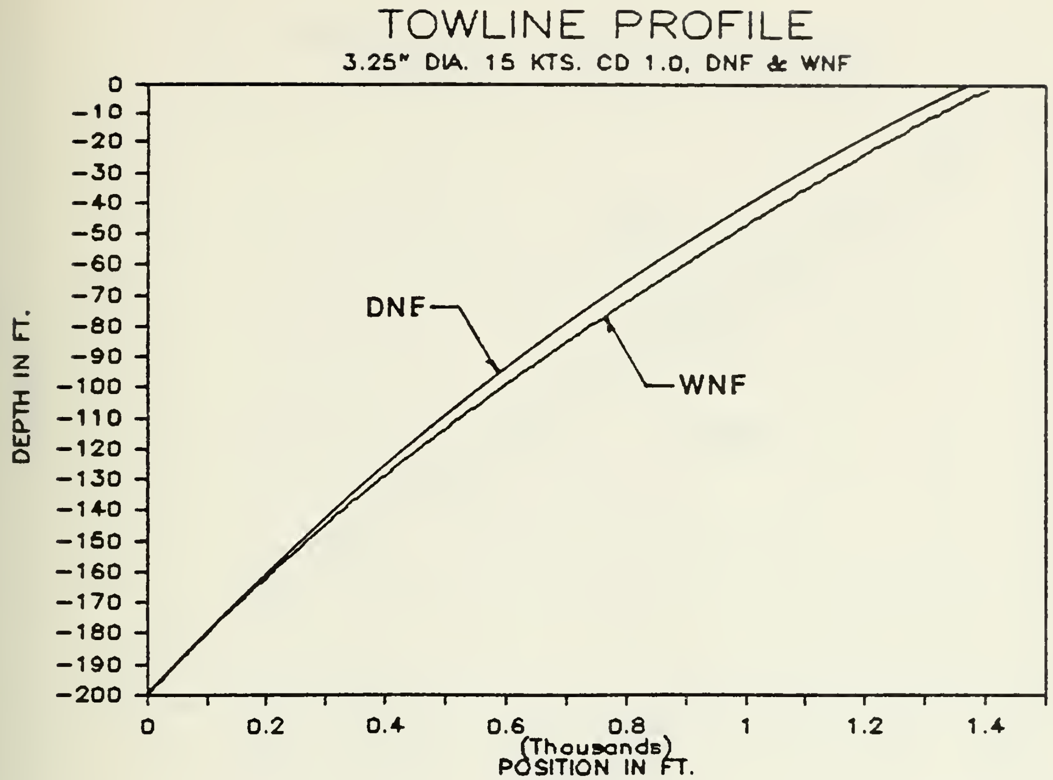


FIGURE C2

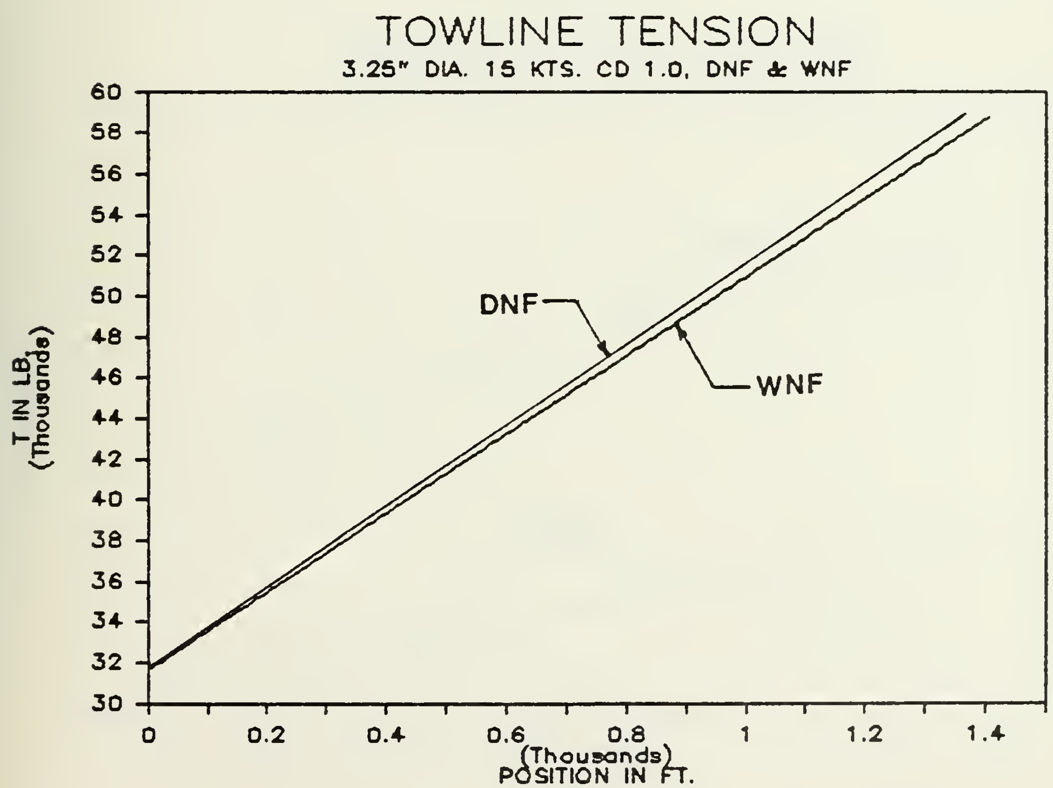


FIGURE C3

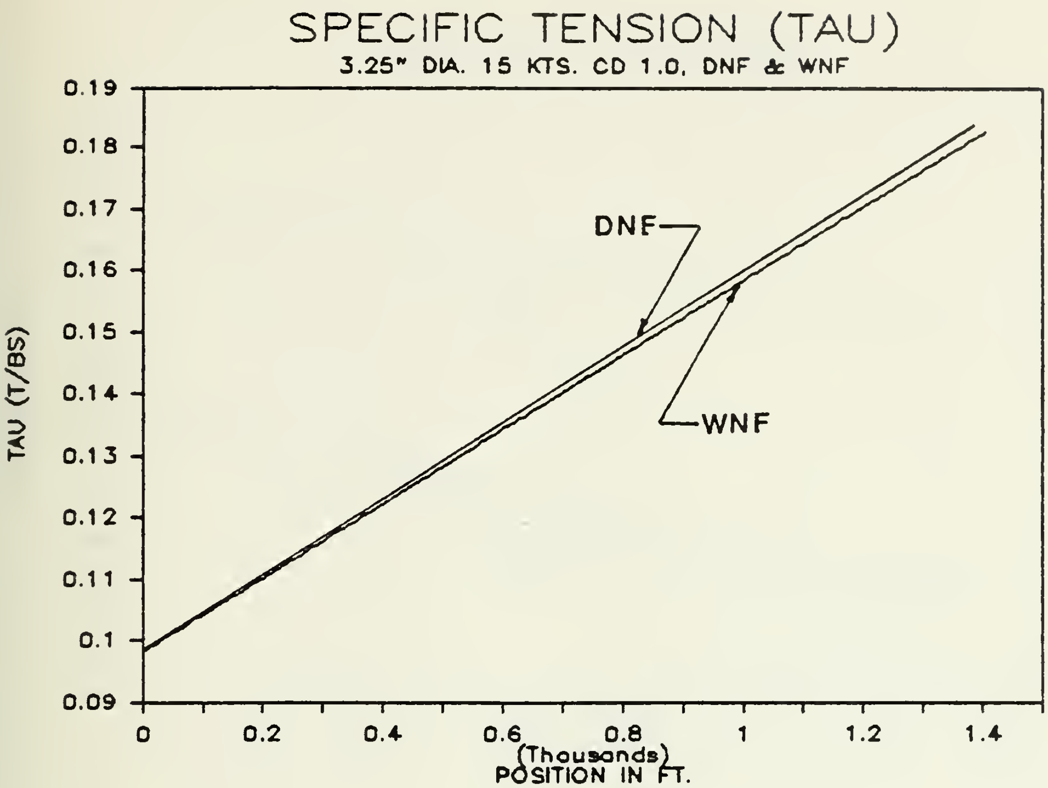


FIGURE C4

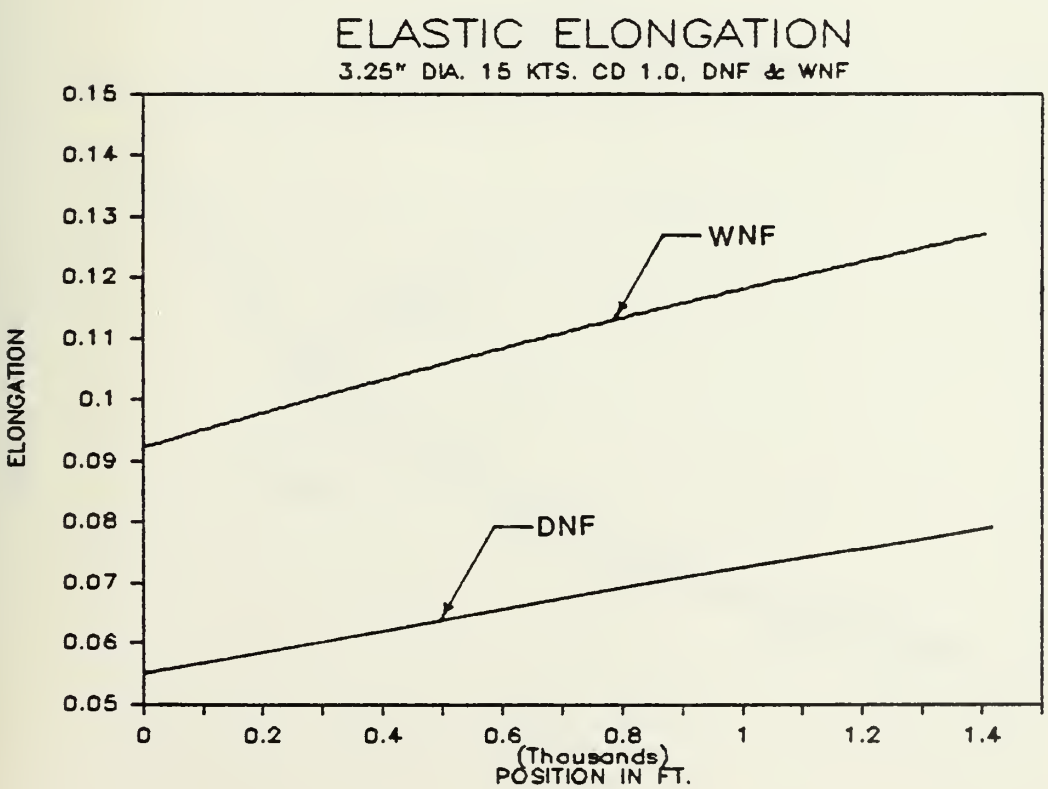


FIGURE C5

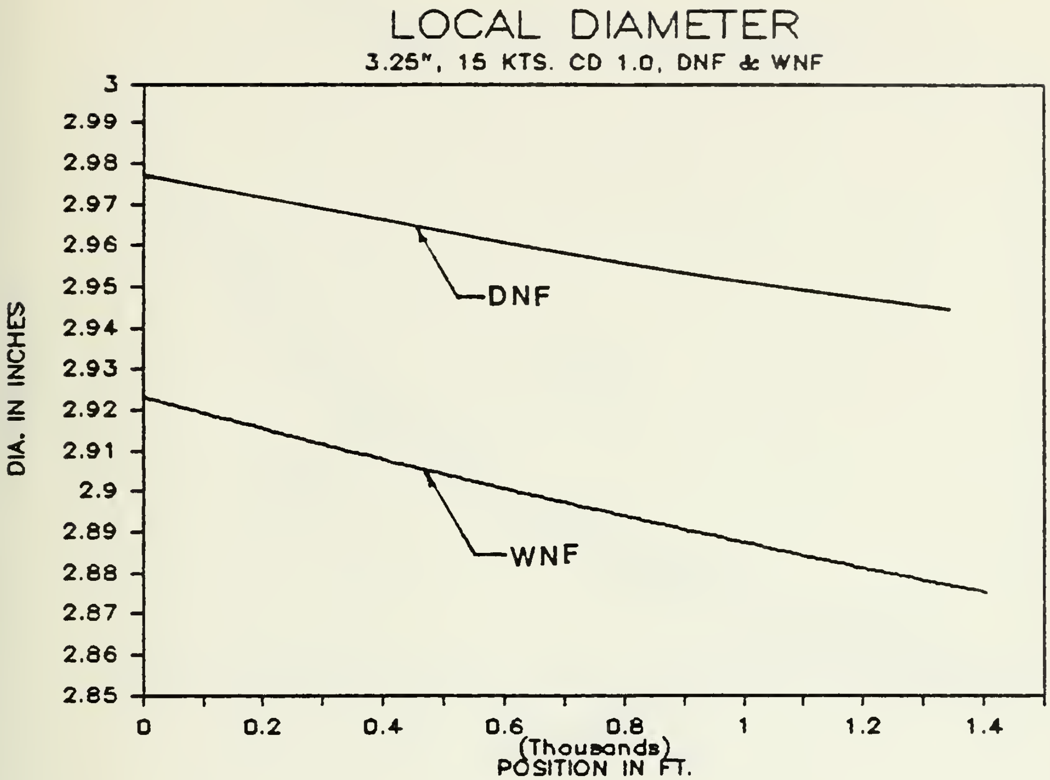
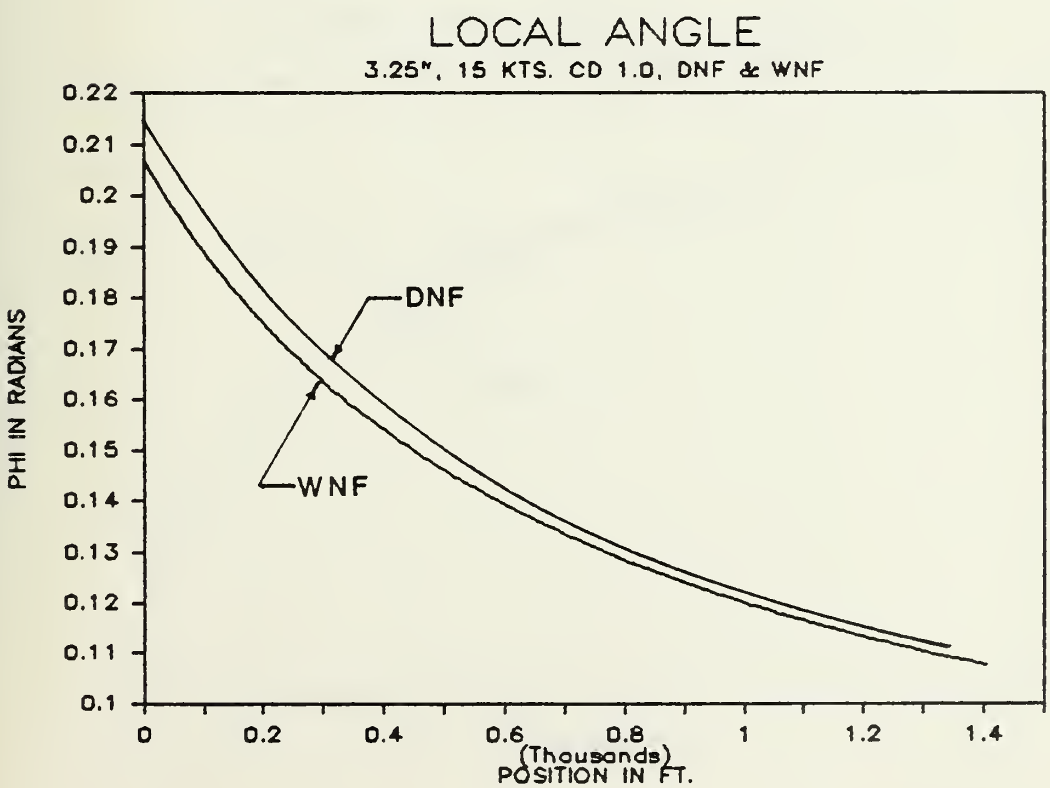


FIGURE C6



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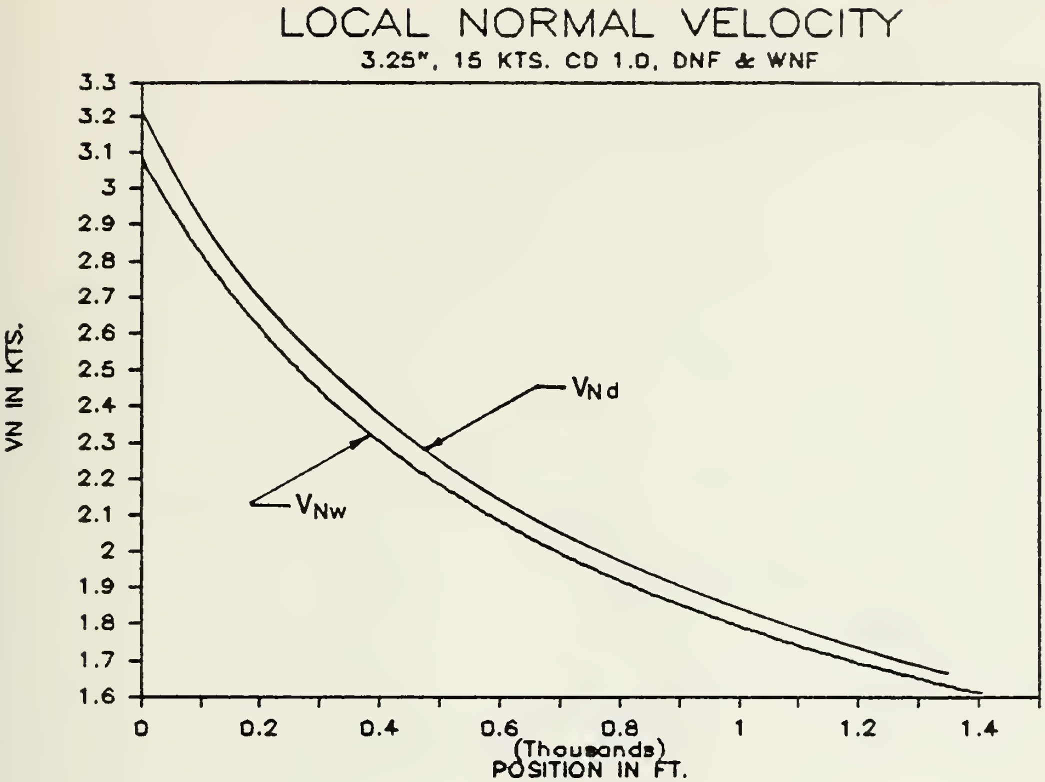

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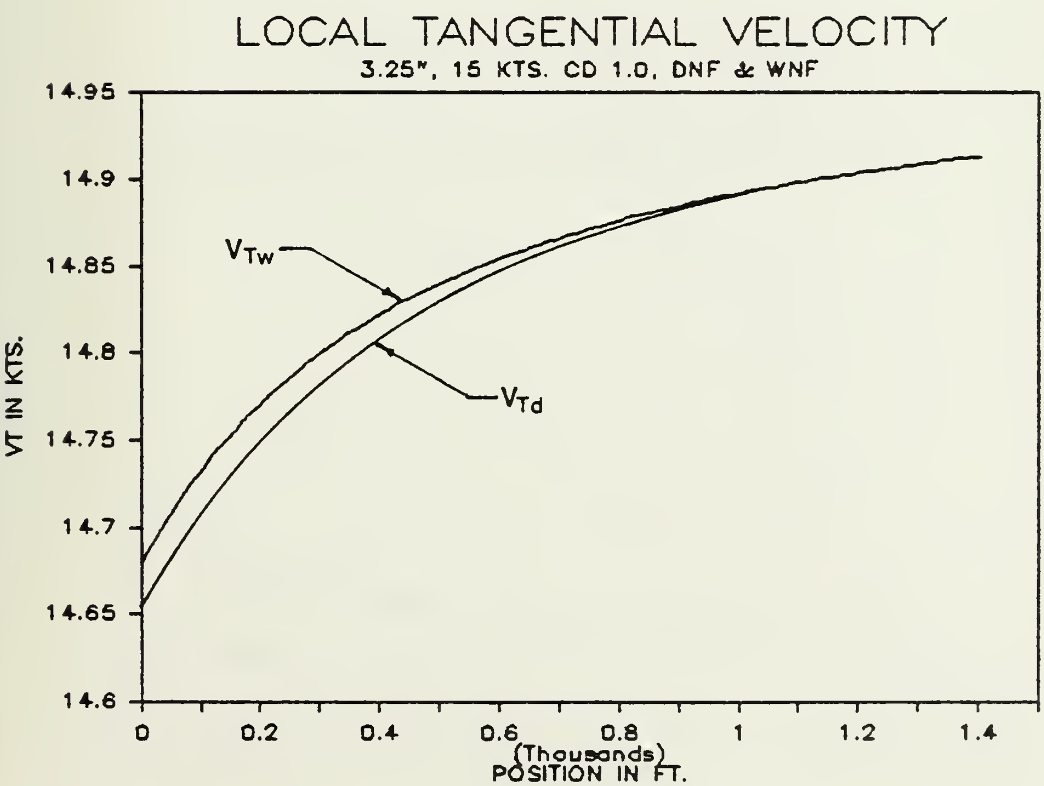

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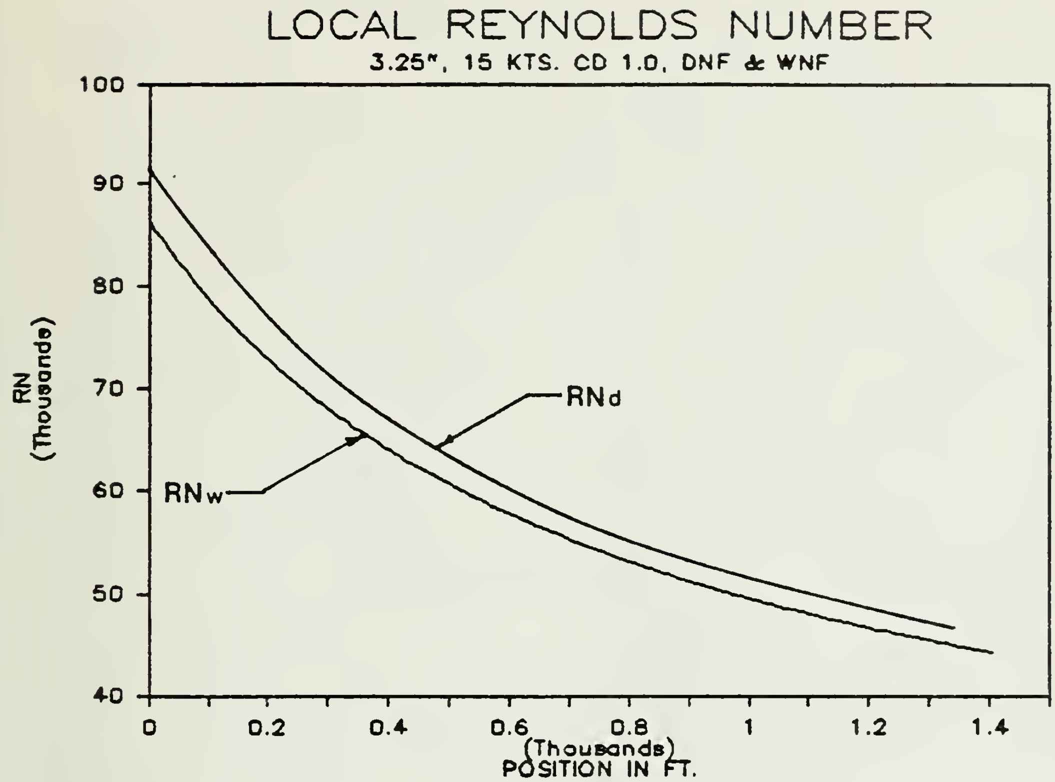


FIGURE C10

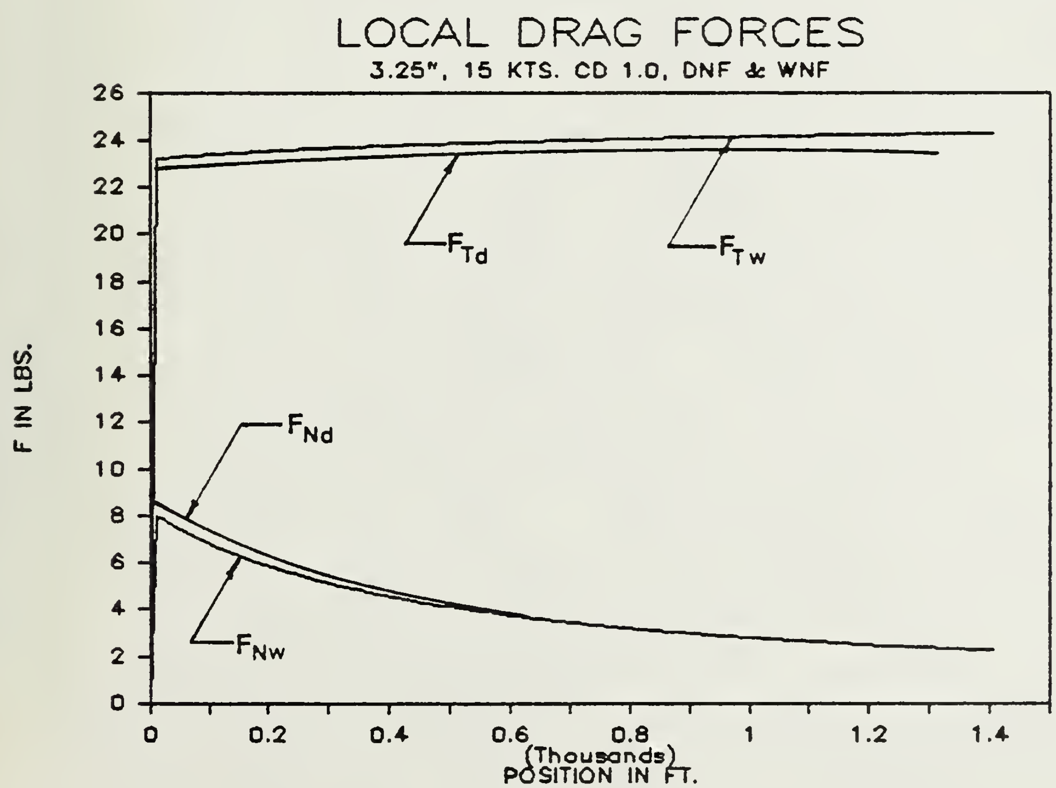
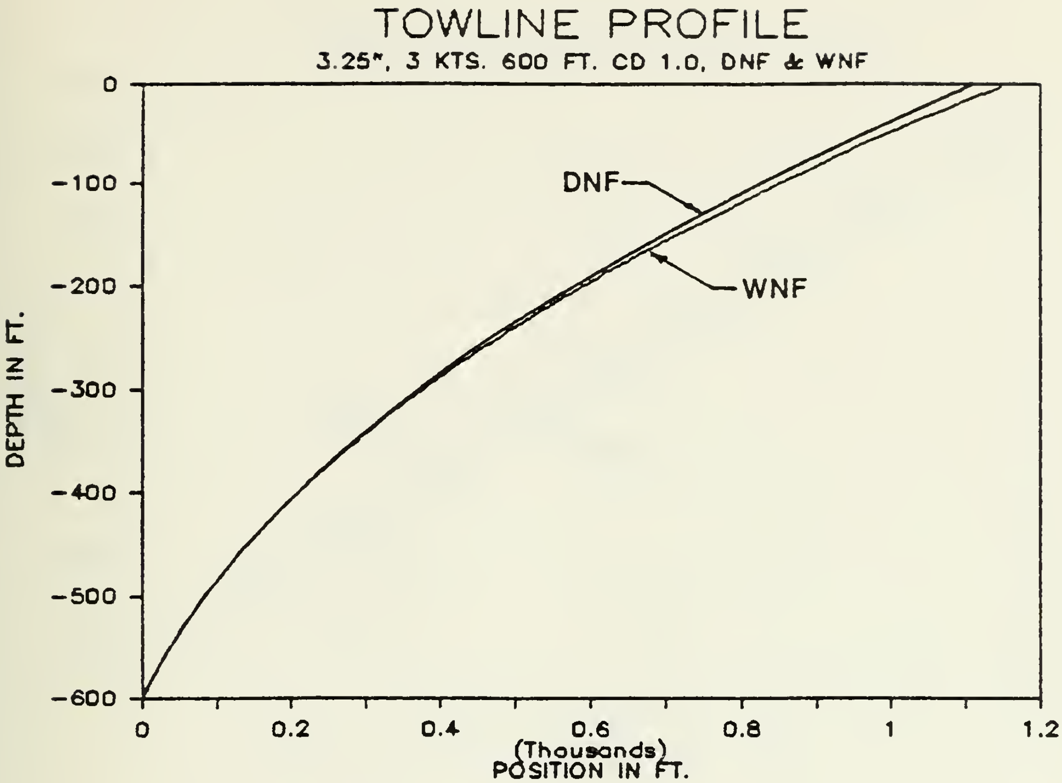


FIGURE C11



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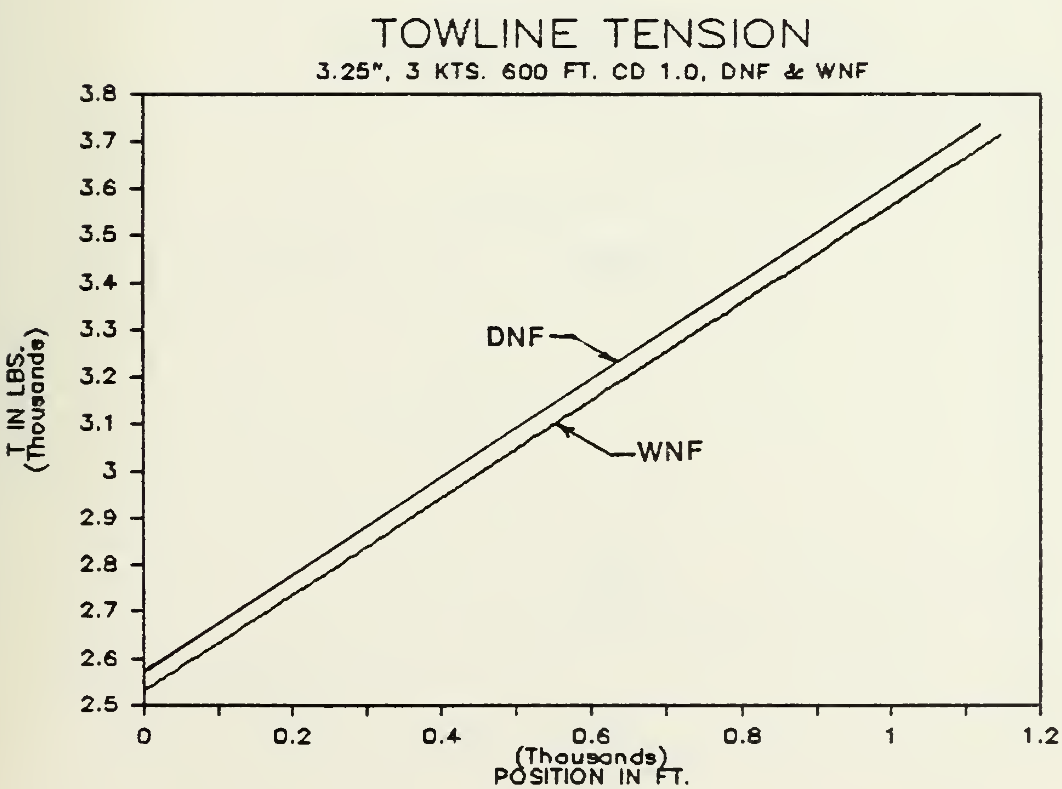

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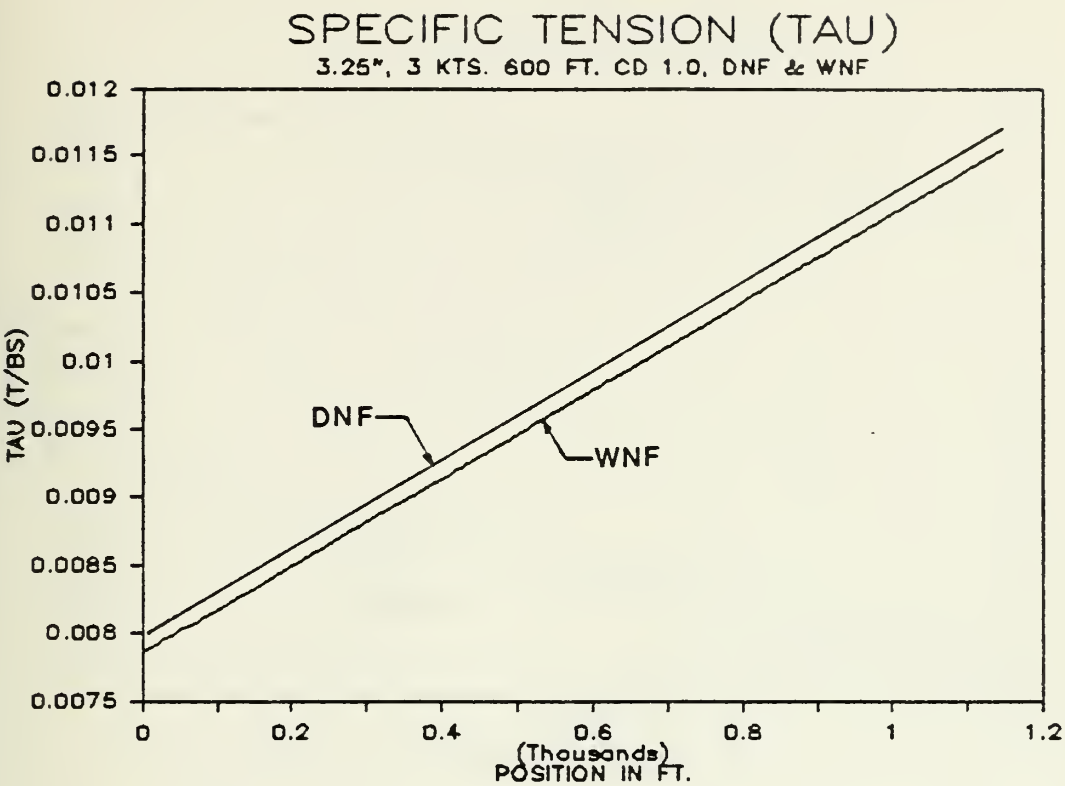

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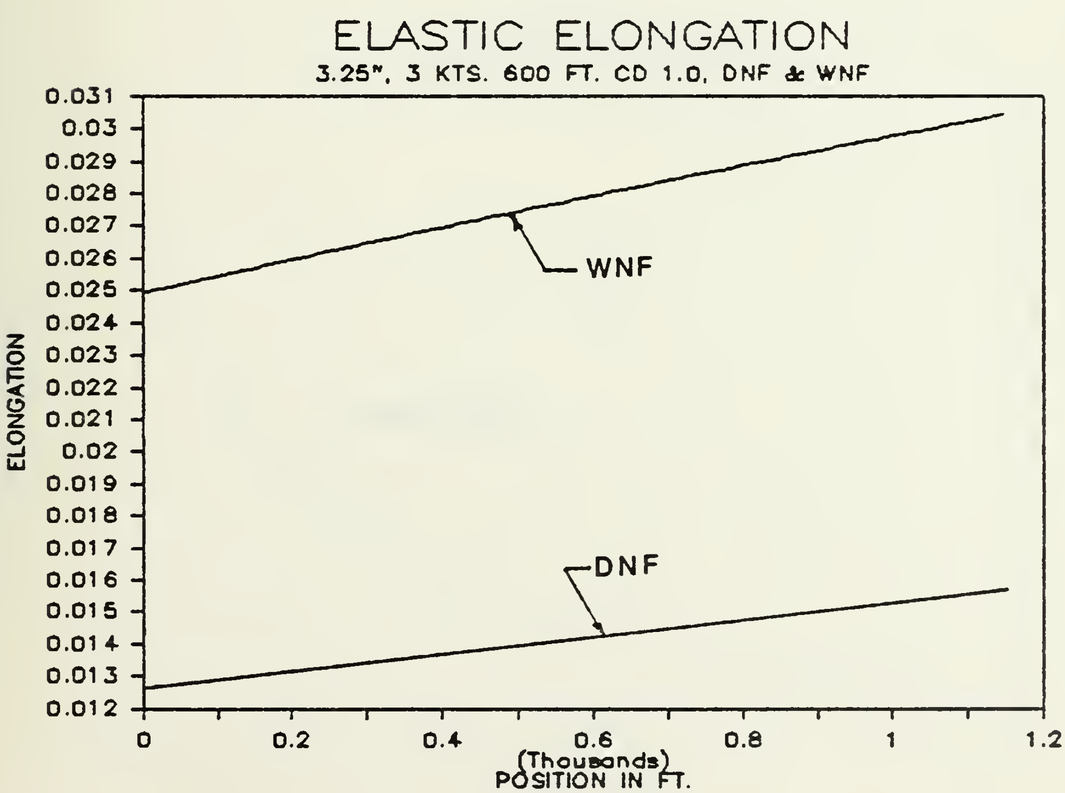

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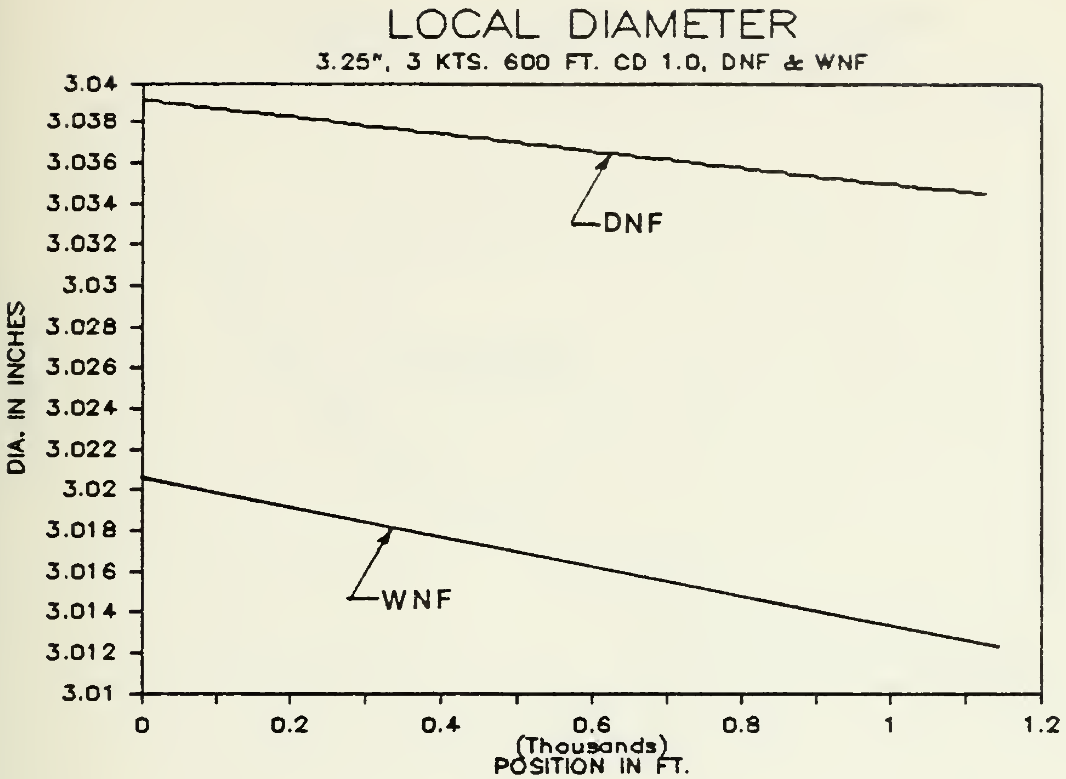

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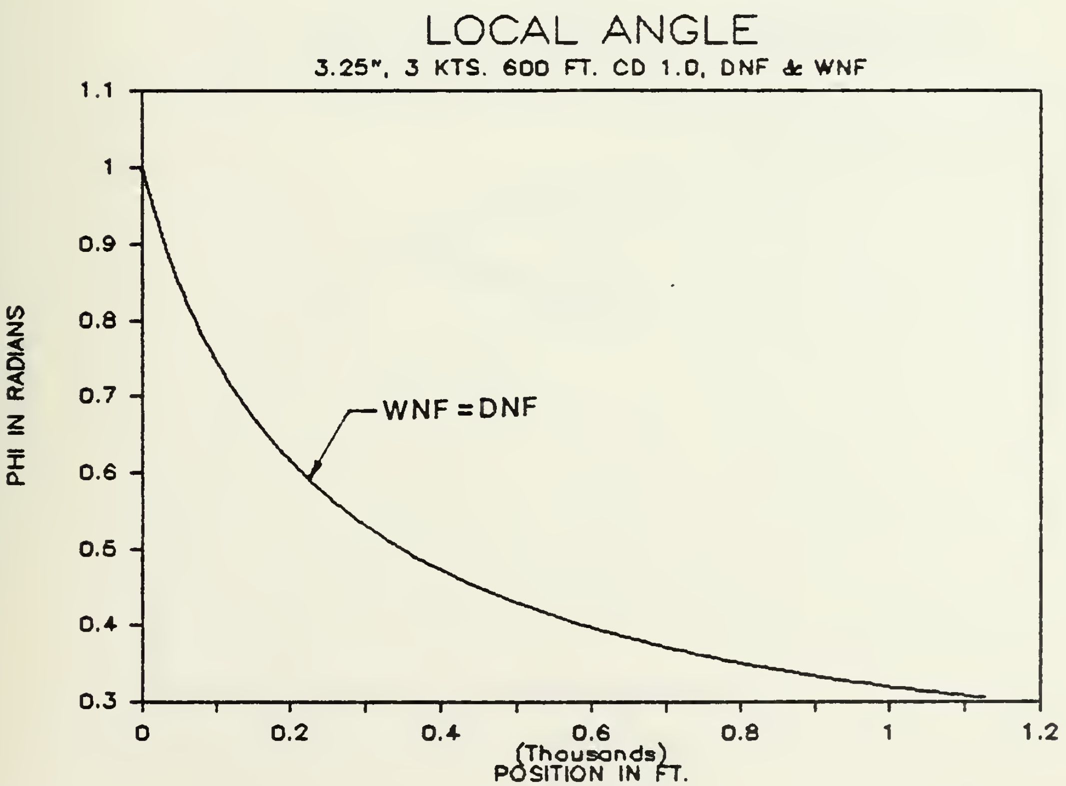

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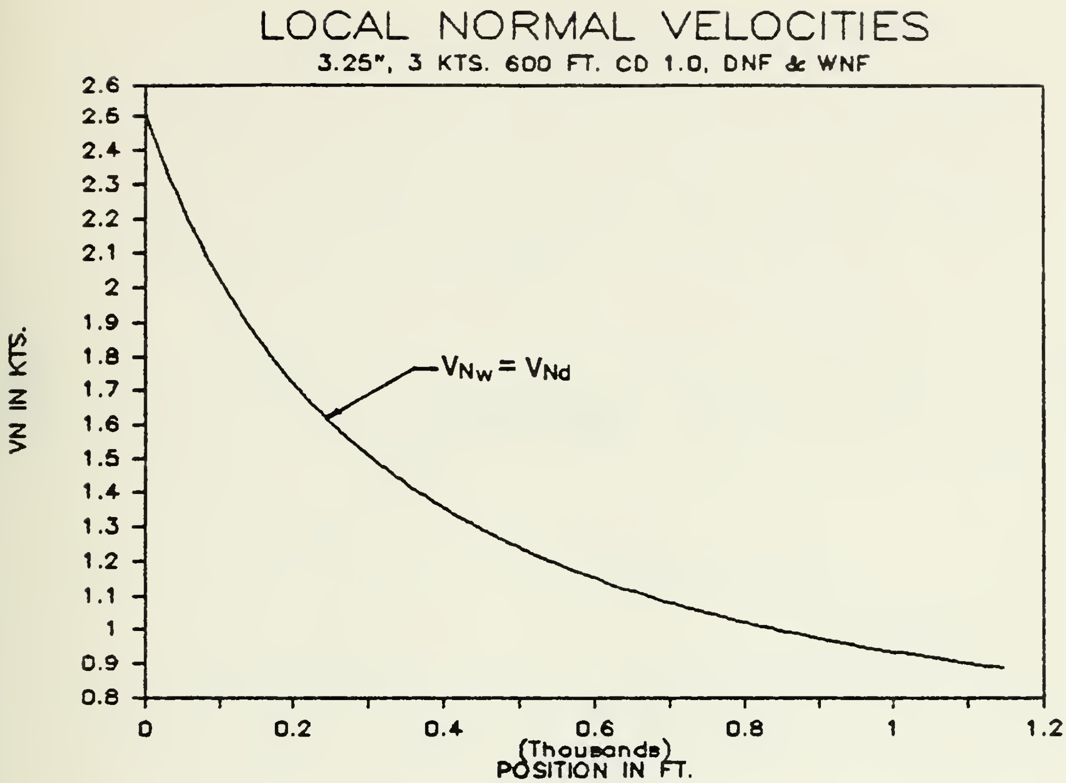

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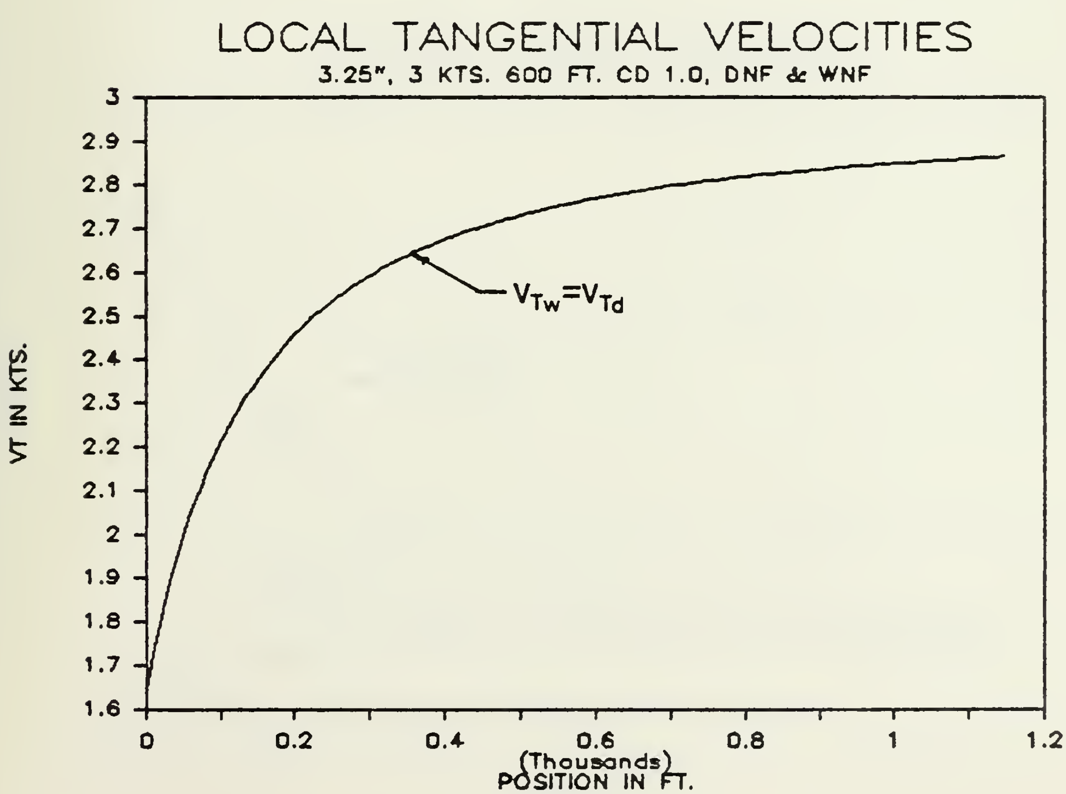

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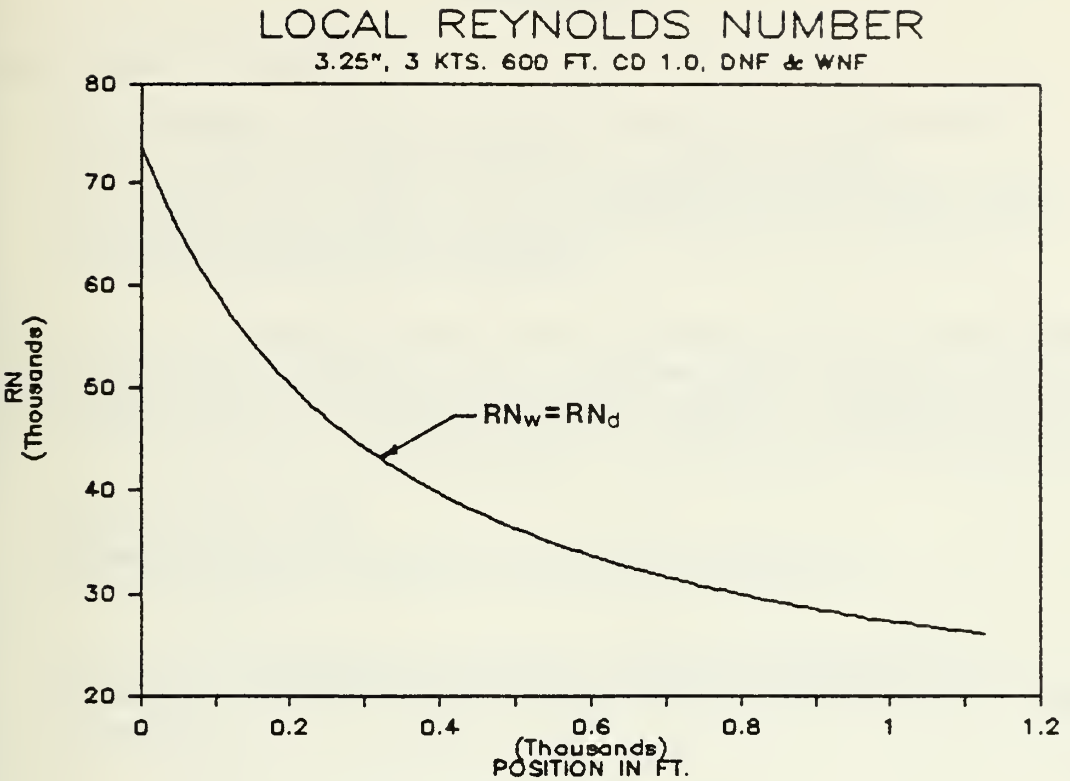
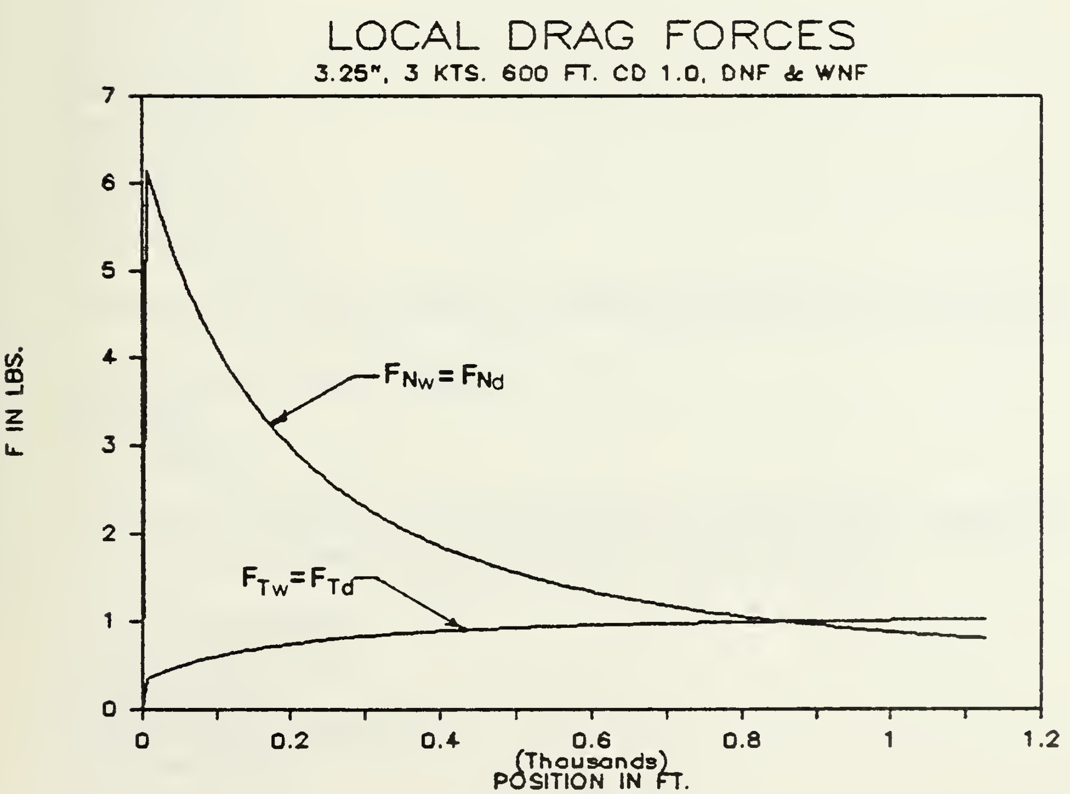


FIGURE C20



APPENDIX D

D1. Design Program

```
C
C      THIS PROGRAM IS USED TO PREDICT THE SIZE OF A DOUBLE BRAID
C      NYLON ROPE WHICH CAN BE USED AS A MARINE TOWLINE
C      THE ALGORITHM IS BASED ON A SUBMARINE TOWING SYSTEM WHERE
C      THE DESIGN POINT IS FOR A SUBMERGED TOWING CONDITION AT A
C      PREDETERMINED DEPTH AND SPEED
C
C      INPUT PARAMETERS CONSIST OF THE GEOMETRY OF THE TOWED VESSEL, THE
C      LENGTH OF THE DESIRED TOWLINE AND THE DESIRED MINIMUM LOAD AS A
C      PERCENTAGE OF THE RATED BREAKING STRENGTH, A MINIMUM VALUE OF 0.08
C      TO 0.1 IS STRONGLY RECOMMENDED TO AVOID POOR OFF-DESIGN BEHAVIOR
C
      IMPLICIT REAL(L-M)
      DIMENSION E(1200),T(1200),PHI(1200),Z(1200),X(1200),Y(250),ST(25
     1 0),DI(1200),TH(1200)
C
C      THE VALUE CHOSEN FOR CD HERE IS 1.0, THIS PARAMETER HAS A
C      SIGNIFICANT EFFECT ON THE UPPER LOAD LIMIT AND SHOULD BE CHOSEN
C      WITH CARE
C
      DATA RHOW,RHOL,PI,CD/2.0,2.209,3.141593,1.0/
      WRITE(*,101)
      READ(*,102) L
      WRITE(*,103)
      READ(*,102) D
      WRITE(*,105)
      READ(*,102) VK
      WRITE(*,104)
      READ(*,102) TD
      WRITE(*,106)
      READ(*,107) IL
      WRITE(*,108)
      READ(*,102) SLMIN
C
C      THE FOLLOWING LINE ASSUME A CONSTANT SHRINKAGE OF 5% FOR NYLON
C
      SHRK=FLOAT(IL)-.05*FLOAT(IL)
      CT=0.04*CD
      IL=INT(SHRK)
C
C      A1 THRU RT PROVIDE AN APPROXIMATION OF THE INITIAL TENSION IN
C      THE TOWLINE FROM THE RESISTANCE OF THE SUBMARINE
C
      A1=(VK*L*1.69)/.0000129
      A2=(ALOG10(A1)-2)**2
      A3=L/D-1.3606
      A4=.075/A2+.0008+.000789/A3
      A5=PI*D**2*A3*A4
```

```
      A6=A5+9+.001*L*D
      DHP=.00872*VK**3*A6
      RT=(DHP*550)/(1.6886*VK)
      WRITE(*,200) RT
C
C     THE INITIAL ANGLES ARE SET EQUAL TO THE MAXIMUM PHYSICALLY
C     POSSIBLE FOR THIS TOWING GEOMETRY
C
      TAU1=SLMIN
      UPHI=PI/2-.01
      LPHI=.01
      DS=1.
15    MPHI=(UPHI+LPHI)/2.
      TAU=TAU1
C
C     INITIAL BREAKING STRENGTH AND DIA  BASED ON SAMSON DATA
C
      BSL=RT/(SLMIN*COS(MPHI))
      DIL=(BSL/34148.5)**.5258
C
C     HYDROSTATIC COMPONENT OF TENSION
C
      TH(1)=.3491*TD*DIL**2
      DO 10 I=1,3
C
C     INITIAL EFFECTIVE TENSION AS A FCN OF ANGLE
C
      IF(I-2) 51,52,53
51    T(1)=RT/COS(LPHI)
      PHI(1)=LPHI
      GO TO 55
52    T(1)=RT/COS(UPHI)
      PHI(1)=UPHI
      GO TO 55
53    T(1)=RT/COS(MPHI)
      PHI(1)=MPHI
C
C     INITIAL ELASTIC ELONGATION FROM DNF
C
55    E(1)=.2119*(TAU1)**.5848
      DI(1)=DIL*.941/(1+E(1)/2)
      Z(1)=0
      X(1)=0
      TE=E(1)
C
C     BEGIN ITERITIVE SOLN OF GOVN  EQN
C
      DO 20 J=2,IL
C
```

```
C       ESTABLISH SIMPSONS MULTIPLIERS FOR TOTAL ELONGATION
C
        IF((2*INT(J/2)).LT.J) TFAC=2
        IF((2*INT(J/2)).EQ.J) TFAC=4
        IF(J.EQ.IL) TFAC=1
        IN=1
C       LOCAL ELAS. ELONG
        E(J)=.2119*(TAU)**.5848
        TE=TE+E(J)*TFAC
C       LOCAL DIA.
        DI(J)=DIL*.941/(1+E(J)/2)

C       LOCAL WEIGHT IN WATER
        WB=PI/4.*(DI(J)/12.)**2*32.2*(1+E(J))*(RHOL-RHOW)*1.13
C       LOCAL NORMAL DRAG FORCE
        FD1=.5*RHOW*CD*(DI(J)/12.)*(VK*1.6886)**2
        FD2=SIN(PHI(J-IN))**2*(1+E(J))*1.13
        FD=FD1*FD2
C       LOCAL TANGENTIAL DRAG FORCE
        FT1=.5*RHOW*CT*PI*(DI(J)/12.)*(VK*1.6886)**2
        FT2=COS(PHI(J-IN))**2*(1+E(J))*1.13
        FT=FT1*FT2
C       LOCAL ANGLE
        PHI(J)=PHI(J-IN)+DS/T(J-IN)*(-FD+WB*COS(PHI(J-IN)))
C       DEPTH OF ELEMENT
        Z(J)=Z(J-IN)+DS*SIN(PHI(J-IN))*(1+E(J))*1.13
C       LOCAL HYDROSTATIC TENSION
        TH(J)=0.3491*(TD-Z(J))*DI(J)**2
C       LOCAL EFFECTIVE TENSION
        T(J)=T(J-IN)+DS*(FT+WB*SIN(PHI(J-IN)))
        TAU=T(J)/BSL
        X(J)=X(J-IN)+DS*COS(PHI(J-IN))*(1+E(J))*1.13
        JM=J
C       CHECK FOR IMPOSSIBLE SOLUTIONS
        IF((Z(J)-TD).GT.10.) GO TO 59
        IF((TD-Z(J)).GT.(2*TD)) GO TO 59
20      CONTINUE
C       CHECK FOR SOLUTION BASED ON FINAL DEPTH AT STRETCHED LENGTH
59      IF(I-2) 61,62,63
61      SZL=Z(JM)
        SXL=X(JM)
        FZL=TD-SZL
C       ERROR CONDITION ON LOWER ANGLE
        IF(FZL.LT.0.) GO TO 1000
        GO TO 10
62      SZU=Z(JM)
        SXU=X(JM)
        FZU=TD-SZU
C       ERROR CONDITION ON UPPER ANGLE
        IF(FZU.GT.0) GO TO 1010
```

```
      GO TO 10
63     SZM=Z(JM)
      SXM=X(JM)
      FZM=TD-SZM
C      CHECK FOR DEPTH SOLN CLOSE TO SURFACE
      IF(ABS(FZM).LT.1) GO TO 90
10    CONTINUE
C      CHECK TO SEE IF LIMITS ARE ON OPPOSITE SIDES OF SOLN
C      IF NOT WRITE ERROR MESSAGE
      IF((FZU*FZL).GT.0.) GO TO 80
      DPHI=UPHI-LPHI
C      ERROR CHECK ON ANGLES
      IF (DPHI.EQ.0) GO TO 95
C      CHECK FOR SOLN BASED ON CONVERGING ANGLES
      IF(DPHI.LT.0.001) GO TO 90
C      MODIFY UPPER OR LOWER ANGLE LIMIT FOR NEXT ITERATION
      IF((FZU*FZM).GT.0) UPHI=MPHI

      IF((FZL*FZM).GT.0) LPHI=MPHI
C      SCREEN WRITE TO MONITOR CONVERGENT BEHAVIOR
      WRITE(*,260) FZL,FZM,FZU,LPHI,MPHI,UPHI
C      START NEXT ITERATION IF NEEDED
      GO TO 15
90    SLMAX=T(JM)/BSL
      TE=(DS/3.*TE+.13*FLOAT(IL))/FLOAT(IL)
C      WRITE FINAL SOLN TO DATA FILE
      OPEN(16,FILE='TOWNYLON.DAT',STATUS='NEW')
      WRITE(16,225) VK,RT,TD,IL,TE
      WRITE(16,220) DIL,BSL,SLMIN,SLMAX
      WRITE(16,230)
      IV=5
      DO 30 K=1,IL,IV
      IF(T(K).LE.0.) GOTO 30
      Y(K)=Z(K)-TD
      ST(K)=T(K)/BSL
      WRITE(16,240) E(K),T(K),TH(K),ST(K),DI(K),PHI(K),Y(K),X(K)
30     CONTINUE
      CLOSE(16)
      GO TO 95
80     WRITE(*,210)
      WRITE(*,260) FZL,FZM,FZU,LPHI,MPHI,UPHI
260    FORMAT( ' FZL ',E9.2,' FZM ',E9.2,' FZU ',E9.2,' LPHI ',E11.4,'
    1  MPHI ',E11.4,' UPHI ',E11.4/)
101   FORMAT( ' INPUT LENGTH OF TOWED VESSEL :')
102   FORMAT( F6.2)
103    FORMAT( ' INPUT DIAMETER OF TOWED VESSEL :')
104    FORMAT( ' INPUT DESIGN DEPTH OF TOW :')
105    FORMAT( ' INPUT DESIRED DESIGN VELOCITY IN KNTS :')
106    FORMAT( ' INPUT LENGTH OF TOWLINE TO THE NEAREST 10 FT,AS AN INT
    1EGER :')
```

```
107     FORMAT( I6)
108     FORMAT( ' INPUT DESIRED MINIMUM SPECIFIC LOADING AT TOWED VESSEL
       1  :')
200     FORMAT( ' RESISTANCE IN LBF :',F10.2)
210     FORMAT( ' ANGLE NOT WITHIN RANGE OF INITIAL ESTIMATE!')
220     FORMAT( ' ROPE DIAMETER IN INCHES ',F6.4,' NEW-DRY BREAKING STR
       1ENGTH LBS :',F10.2/ ' MINIMUM SPECIFIC LOAD ',F6.3,' MAXIMUM SPEC
       2IFIC LOAD :',F6.3)
225     FORMAT( ' TOW VELOCITY   ',F6.1,' RESISTANCE ',F8.1,' TOW DEPTH
       1',F6.1/' TOWLINE LENGTH :',I6,' TOTAL ELONGATION ',F8.3)
230     FORMAT( ' ELONGATION  TENSION    THYDRO      TAU       DIA     A
       1NGLE     DEPTH       REACH')
240     FORMAT( ' ',F6.5,4X,F10.3,1X,F10.3,2X,F6.5,4X,F6.4,3X,F6.4,2X,F8
       1.3,3X,F9.3)
250     FORMAT( ' ',2F6.2)
        GO TO 95
1000    WRITE(*,270) FZL
270     FORMAT( ' LOWER LIMIT IS ABOVE WATERLINE! ',E14.6)
        GO TO 95
1010    WRITE(*,280) FZU
280     FORMAT( ' UPPPER LIMIT IS BELOW WATERLINE! ',E14.6)
95      END
```

D2. Analysis Program

```
C       THIS PROGRAM IS USED TO ANALYSE THE OFF-DESIGN PERFORMANCE
C       OF A PRE-SIZED NYLON TOWLINE, IT CALCULATES BOTH THE DNF AND
C       WNF LIMITS FOR HIGH CYCLE AND NEW BEHAVIOR RESPECTIVELY
C
C       THERE ARE TWO DATA FILES, ONE FOR DNF BEHAVIOR AND ONE FOR
C       WNF BEHAVIOR, GENERAL DATA IS INCLUDED AS A HEADER IN THE
C       WNF OUTPUT FILE
C
C       THE DRAG OF THE VESSEL BEING TOWED MUST BE KNOWN AND PROVIDED
C       AS INPUT TO THE PROGRAM
C
C       THE OUTPUT INCLUDES THE LOCAL REYNOLDS NUMBER, NORMAL AND
C       TANGENTIAL VELOCITIES FOR LOCAL FLOW CHARACTERISTICS
C
        IMPLICIT REAL(L-M)
        DIMENSION E(2,1200),T(2,1200),PHI(2,1200),Z(2,1200),X(2,1200)
        DIMENSION Y(2,250),ST(2,250),DI(2,1200),SLMAX(2),TE(2)
        DIMENSION FD(2,1200),FT(2,1200),VN(2,1200),VT(2,1200),RN(2,1200)
        DIMENSION TH(2,1200)
        DATA RHOW,RHOL,PI/2.0,2.209,3.141593/
        WRITE(*,101)
```

```
      READ(*,102) DIL
      WRITE(*,103)
      READ(*,102) BSL
      WRITE(*,105)
      READ(*,102) VK
      WRITE(*,104)
      READ(*,102) TD
      WRITE(*,106)
      READ(*,107) IL
      WRITE(*,108)
      READ(*,102) RT
      WRITE(*,109)
      READ(*,102) CD
      CT=0.04*CD
      SHRK=FLOAT(IL)-.05*FLOAT(IL)
      IL=INT(SHRK)
      FD(1,1)=0
      FD(2,1)=0
      FT(1,1)=0
      FT(2,1)=0
      DO 25 IJ=1,2
      UPHI=PI/2.-.01
      LPHI=.01-PI/2
      DS=1.
      TH(1,1)=.3491*TD*DIL**2
      TH(2,1)=TH(1,1)
15    MPHI=(UPHI+LPHI)/2
      DO 10 I=1,3
      IF(I-2) 51,52,53
51    T(IJ,1)=RT/COS(LPHI)
      PHI(IJ,1)=LPHI
      GO TO 55
52    T(IJ,1)=RT/COS(UPHI)

      PHI(IJ,1)=UPHI
      GO TO 55
53    T(IJ,1)=RT/COS(MPHI)
      PHI(IJ,1)=MPHI
55    TAU1=T(IJ,1)/BSL
      TAU=TAU1
      IF(IJ.EQ.1) E(IJ,1)=.307*(TAU1)**.518
      IF(IJ.EQ.2) E(IJ,1)=.2119*(TAU1)**.5848
      DI(IJ,1)=DIL*.941/(1+E(IJ,1)/2)
      Z(IJ,1)=0.
      X(IJ,1)=0
      TE(IJ)=E(IJ,1)
      DO 20 J=2,IL
      IF((2*INT(J/2)).LT.J) TFAC=2.
      IF((2*INT(J/2)).EQ.J) TFAC=4
      IF(J.EQ.IL) TFAC=1.
```

```
      IN=1
      IF(IJ.EQ.1) E(IJ,J)=.307*(TAU)**.518
      IF(IJ.EQ.2) E(IJ,J)=.2119*(TAU)**.5848
      TE(IJ)=TE(IJ)+E(IJ,J)*TFAC
      DI(IJ,J)=DIL*.941/(1+E(IJ,J)/2)
      WB=PI/4.*(DI(IJ,J)/12.)**2*32.2*(1+E(IJ,J))*(RHOL-RHOW)*1.13
      FD1=.5*RHOW*CD*(DI(IJ,J)/12.)*(VK*1.6886)**2
      FD2=SIN(PHI(IJ,J-IN))**2*(1+E(IJ,J))*1.13
      FD(IJ,J)=FD1*FD2
      FT1=.5*RHOW*CT*PI*(DI(IJ,J)/12.)*(VK*1.6886)**2
      FT2=COS(PHI(IJ,J-IN))**2*(1+E(IJ,J))*1.13
      FT(IJ,J)=FT1*FT2
      PHI(IJ,J)=PHI(IJ,J-IN)+DS/T(IJ,J-IN)*(-FD(IJ,J)+WB*
     1COS(PHI(IJ,J-IN)))
      Z(IJ,J)=Z(IJ,J-IN)+DS*SIN(PHI(IJ,J-IN))*(1+E(IJ,J))*1.13
      TH(IJ,J)=.3491*(TD-Z(IJ,J))*DI(IJ,J)**2
      T(IJ,J)=T(IJ,J-IN)+DS*(FT(IJ,J)+WB*SIN(PHI(IJ,J-IN)))
      TAU=T(IJ,J)/BSL
      X(IJ,J)=X(IJ,J-IN)+DS*COS(PHI(IJ,J-IN))*(1+E(IJ,J))*1.13
      JM=J
      IF((Z(IJ,J)-TD).GT.10.) GO TO 59
      IF((TD-Z(IJ,J)).GT.(2*TD)) GO TO 59
20    CONTINUE
59    IF(I-2) 61,62,63
61    SZL=Z(IJ,JM)
      SXL=X(IJ,JM)
      FZL=TD-SZL
      IF(FZL.LT.0.) GO TO 1000
      GO TO 10
62    SZU=Z(IJ,JM)
      SXU=X(IJ,JM)
      FZU=TD-SZU
      IF(FZU.GT.0) GO TO 1010
      GO TO 10
63    SZM=Z(IJ,JM)
      SXM=X(IJ,JM)
      FZM=TD-SZM
      IF(ABS(FZM).LT.0.1) GO TO 90
10    CONTINUE

      IF((FZU*FZL).GT.0.) GO TO 80
      DPHI=UPHI-LPHI
      IF (DPHI.EQ.0.) GO TO 95
      IF(DPHI.LT.0.0001) GO TO 90
      IF((FZU*FZM).GT.0.) UPHI=MPHI
      IF((FZL*FZM).GT.0.) LPHI=MPHI
      WRITE(*,260) FZL,FZM,FZU,LPHI,MPHI,UPHI
      GO TO 15
90    SLMAX(IJ)=T(IJ,JM)/BSL
      SLMIN=T(IJ,1)/BSL
```

```
      TB(IJ)=(DS/3 *TB(IJ)+ 13*FLOAT(IL))/FLOAT(IL)
25    CONTINUE
      DO 35 II=1,2
      DO 45 JJ=1,IL
      VN(II,JJ)=VK*SIN(PHI(II,JJ))
      VT(II,JJ)=VK*COS(PHI(II,JJ))
      RN(II,JJ)=(VN(II,JJ)*1 6889*DI(II,JJ))/(1 47E-5*12)
45    CONTINUE
35    CONTINUE
      OPEN(16,FILE='TOWANW DAT',STATUS='NEW')
      OPEN(17,FILE='TOWAND DAT',STATUS='NEW')
      OPEN(18,FILE='FVRW DAT',STATUS='NEW')
      OPEN(19,FILE='FVRD DAT',STATUS='NEW')
      WRITE(16,225) VK,RT,TD,IL,TB(1),TB(2),CD
      WRITE(16,220) DIL,BSL,SLMIN,SLMAX(1),SLMAX(2)
      WRITE(18,225)VK,RT,TD,IL,TB(1),TB(2),CD
      WRITE(18,220)DIL,BSL,SLMIN,SLMAX(1),SLMAX(2)
      WRITE(18,232)
      WRITE(19,238)
      WRITE(16,230)
      WRITE(17,235)
      IV=10
      DO 30 K=1,IL,IV
      Y(1,K)=Z(1,K)-TD
      Y(2,K)=Z(2,K)-TD
      ST(1,K)=T(1,K)/BSL
      ST(2,K)=T(2,K)/BSL
      WRITE(16,240) E(1,K),T(1,K),TH(1,K),ST(1,K),DI(1,K),PHI(1,K),Y(
     11,K),X(1,K)
      WRITE(18,242) VN(1,K),VT(1,K),RN(1,K),FD(1,K),FT(1,K)
      WRITE(17,245) E(2,K),T(2,K),TH(2,K),ST(2,K),DI(2,K),PHI(2,K),Y(
     12,K),X(2,K)
      WRITE(19,242) VN(2,K),VT(2,K),RN(2,K),FD(2,K),FT(2,K)
30    CONTINUE
      CLOSE(16)
      GO TO 95
80    WRITE(*,210)
      WRITE(*,260) FZL,FZM,FZU,LPHI,MPHI,UPHI
260   FORMAT( ' FZL ',E9.2,' FZM ',E9.2,' FZU ',E9.2,' LPHI ',E11 4,'
     1 MPHI ',E11 4,' UPHI ',E11.4/)
101   FORMAT( ' INPUT DIAMETER OF TOWLINE :')
102   FORMAT( F10.3)
103   FORMAT( ' INPUT NDBS OF TOWLINE :')
104   FORMAT( ' INPUT DESIGN DEPTH OF TOW :')
105   FORMAT( ' INPUT DESIRED DESIGN VELOCITY IN KNTS :')

106   FORMAT( ' INPUT LENGTH OF TOWLINE TO THE NEAREST 10 FT,AS AN INT
     1EGER :')
107   FORMAT( I6)
108   FORMAT( ' INPUT RESISTANCE OF TOWED VESSEL :')
```

```
109     FORMAT( ' INPUT DESIRED DRAG COEFFICIENT, CD ')
210     FORMAT( ' ANGLE NOT WITHIN RANGE OF INITIAL ESTIMATE!')
220     FORMAT( ' ROPE DIA IN INCHES :',F6.4,' NEW-DRY BREAKING STRENGTH
       1 LBS :',F10.2/ ' MINIMUM SPECIFIC LOAD :',F6.3/' WE MAXIMUM SPECIF
       2IC LOAD :',F6.3,' DE MAXIMUM SPECIFIC LOAD :',F6.3)
225     FORMAT( ' TOW VELOCITY : ',F6.1,' RESISTANCE :',F8.1,' TOW DEPTH
       1:',F6.1/' TOWLINE LENGTH :',I6,' WE TOTAL ELONGATION :',F8.3,' DE
       2TOTAL ELONGATION :',F8.3,' NORMAL DRAG COEFFICIENT :',F8.4)
230     FORMAT( ' WE ELONG     WE TEN     WE TH      WE TAU     WE DIA   WE A
       1NGLE   WE DEPTH     WE REACH')
232     FORMAT( ' ',' VNW     VTW         RN        FDW      FTW  ')
235     FORMAT( ' DE ELONG     DE TEN     WE TH      DE TAU     DE DIA   DE A
       1NGLE   DE DEPTH      DE REACH')
238     FORMAT( ' ',' VND     VTW         RN        FDD      FTD  ')
240     FORMAT( ' ',F6.5,4X,F10.3,1X,F8.3,3X,F6.5,4X,F6.4,3X,F6.4,2X,F8
       1.3,4X,F9.3)
242     FORMAT( ' ',F6.3,2X,F6.3,2X,E10.3,1X,F7.3,1X,F7.3)
245     FORMAT( ' ',F6.5,4X,F10.3,1X,F8.3,3X,F6.5,4X,F6.4,3X,F6.4,2X,F8
       1.3,4X,F9.3)
        GO TO 95
1000    WRITE(*,270) FZL
270     FORMAT( ' LOWER LIMIT IS ABOVE WATERLINE! ',E14.6)
        GO TO 95
1010    WRITE(*,280) FZU
280     FORMAT( ' UPPPER LIMIT IS BELOW WATERLINE! ',E14.6)
95      END
```

ANALYSIS PROGRAM OUTPUT

TYPE AW122015.DAT

TOW VELOCITY :   15.0 RESISTANCE : 30970.0 TOW DEPTH: 200.0
TOWLINE LENGTH : 1140 WE TOTAL ELONGATION :   0.24
97 NORMAL DRAG COEFFICIENT : 1.0000 DE TOTAL ELONGATION 0.197
ROPE DIA IN INCHES :3.2500 NEW-DRY BREAKING STRENGTH LBS : 322000.00
MINIMUM SPECIFIC LOAD : 0.099
WE MAXIMUM SPECIFIC LOAD : 0.183 DE MAXIMUM SPECIFIC LOAD : 0.182

WE ELONG	WE TEN	WE TAU	WE DIA	WE ANGLE	WE DEPTH	WE REACH
.09231	31646.256	.09828	2.9233	0.2071	-200.000	0.000
.09262	31879.342	.09900	2.9229	0.2047	-197.475	12.084
.09297	32112.682	.09973	2.9224	0.2023	-194.978	24.178
.09332	32346.270	.10045	2.9219	0.2001	-192.508	36.281
.09367	32580.098	.10118	2.9214	0.1978	-190.064	48.394
.09402	32814.160	.10191	2.9209	0.1957	-187.647	60.516
.09437	33048.441	.10263	2.9204	0.1936	-185.254	72.646
.09472	33282.941	.10336	2.9200	0.1916	-182.885	84.786
.09506	33517.648	.10409	2.9195	0.1896	-180.540	96.934
.09541	33752.551	.10482	2.9190	0.1877	-178.217	109.091
.09575	33987.648	.10555	2.9185	0.1859	-175.917	121.255
.09609	34222.941	.10628	2.9180	0.1841	-173.638	133.428
.09643	34458.422	.10701	2.9176	0.1823	-171.381	145.608
.09678	34694.082	.10775	2.9171	0.1806	-169.144	157.797
.09712	34929.914	.10848	2.9166	0.1789	-166.926	169.992
.09746	35165.918	.10921	2.9162	0.1773	-164.729	182.196
.09779	35402.082	.10994	2.9157	0.1757	-162.550	194.406
.09813	35638.406	.11068	2.9152	0.1742	-160.390	206.624
.09847	35874.887	.11141	2.9147	0.1727	-158.247	218.848
.09881	36111.516	.11215	2.9143	0.1712	-156.122	231.080
.09914	36348.293	.11288	2.9138	0.1698	-154.015	243.318
.09948	36585.215	.11362	2.9133	0.1684	-151.924	255.564
.09981	36822.277	.11435	2.9129	0.1670	-149.849	267.815
.10014	37059.477	.11509	2.9124	0.1657	-147.790	280.074
.10047	37296.813	.11583	2.9120	0.1644	-145.747	292.338
.10080	37534.281	.11657	2.9115	0.1631	-143.719	304.609
.10113	37771.871	.11730	2.9110	0.1618	-141.706	316.886
.10146	38009.586	.11804	2.9106	0.1606	-139.708	329.170
.10179	38247.422	.11878	2.9101	0.1594	-137.723	341.459
.10212	38485.379	.11952	2.9097	0.1583	-135.753	353.755
.10245	38723.453	.12026	2.9092	0.1571	-133.796	366.056
.10277	38961.641	.12100	2.9088	0.1560	-131.853	378.363
.10310	39199.938	.12174	2.9083	0.1549	-129.922	390.676
.10342	39438.344	.12248	2.9079	0.1538	-128.005	402.995
.10375	39676.859	.12322	2.9074	0.1528	-126.099	415.319
.10407	39915.480	.12396	2.9070	0.1518	-124.206	427.649
.10439	40154.203	.12470	2.9065	0.1508	-122.326	439.984
.10471	40393.031	.12544	2.9061	0.1498	-120.456	452.325
.10503	40631.957	.12619	2.9057	0.1488	-118.599	464.671
.10535	40870.980	.12693	2.9052	0.1479	-116.752	477.023
.10567	41110.102	.12767	2.9048	0.1469	-114.917	489.380
.10599	41349.313	.12841	2.9043	0.1460	-113.092	501.742

ANALYSIS PROGRAM OUTPUT

.10631	41588.617	.12916	2.9039	0.1451	-111.278	514.110
.10663	41828.012	.12990	2.9035	0.1443	-109.475	526.482
.10694	42067.496	.13064	2.9030	0.1434	-107.682	538.860
.10726	42307.070	.13139	2.9026	0.1426	-105.899	551.242
.10757	42546.730	.13213	2.9022	0.1417	-104.125	563.630
.10788	42786.473	.13288	2.9017	0.1409	-102.362	576.023
.10820	43026.305	.13362	2.9013	0.1401	-100.608	588.420
.10851	43266.215	.13437	2.9009	0.1393	-98.863	600.823
.10882	43506.207	.13511	2.9004	0.1386	-97.128	613.230
.10913	43746.281	.13586	2.9000	0.1378	-95.401	625.642
.10944	43986.434	.13660	2.8996	0.1371	-93.683	638.059
.10975	44226.664	.13735	2.8992	0.1363	-91.974	650.481
.11006	44466.973	.13810	2.8987	0.1356	-90.274	662.907
.11037	44707.352	.13884	2.8983	0.1349	-88.582	675.338
.11068	44947.809	.13959	2.8979	0.1342	-86.898	687.774
.11098	45188.336	.14034	2.8975	0.1335	-85.222	700.214
.11129	45428.938	.14108	2.8970	0.1328	-83.555	712.659
.11159	45669.613	.14183	2.8966	0.1322	-81.895	725.108
.11190	45910.355	.14258	2.8962	0.1315	-80.243	737.562
.11220	46151.168	.14333	2.8958	0.1309	-78.598	750.020
.11250	46392.051	.14407	2.8954	0.1303	-76.961	762.483
.11281	46633.004	.14482	2.8950	0.1296	-75.332	774.950
.11311	46874.020	.14557	2.8946	0.1290	-73.709	787.421
.11341	47115.102	.14632	2.8941	0.1284	-72.094	799.897
.11371	47356.254	.14707	2.8937	0.1278	-70.486	812.377
.11401	47597.473	.14782	2.8933	0.1273	-68.884	824.862
.11431	47838.746	.14857	2.8929	0.1267	-67.290	837.351
.11461	48080.090	.14932	2.8925	0.1261	-65.702	849.844
.11491	48321.496	.15007	2.8921	0.1256	-64.121	862.341
.11520	48562.961	.15082	2.8917	0.1250	-62.546	874.843
.11550	48804.488	.15157	2.8913	0.1245	-60.978	887.348
.11580	49046.074	.15232	2.8909	0.1239	-59.416	899.858
.11609	49287.727	.15307	2.8905	0.1234	-57.860	912.372
.11639	49529.430	.15382	2.8901	0.1229	-56.310	924.890
.11668	49771.199	.15457	2.8897	0.1224	-54.766	937.412
.11697	50013.020	.15532	2.8893	0.1219	-53.229	949.939
.11727	50254.906	.15607	2.8889	0.1214	-51.697	962.469
.11756	50496.844	.15682	2.8885	0.1209	-50.171	975.003
.11785	50738.840	.15757	2.8881	0.1204	-48.650	987.542
.11814	50980.891	.15833	2.8877	0.1199	-47.136	1000.084
.11843	51222.996	.15908	2.8873	0.1195	-45.626	1012.630
.11872	51465.160	.15983	2.8869	0.1190	-44.123	1025.181
.11901	51707.375	.16058	2.8865	0.1185	-42.624	1037.735
.11930	51949.645	.16133	2.8861	0.1181	-41.131	1050.293
.11959	52191.969	.16209	2.8857	0.1176	-39.643	1062.855
.11987	52434.340	.16284	2.8853	0.1172	-38.161	1075.421
.12016	52676.773	.16359	2.8849	0.1168	-36.683	1087.991
.12045	52919.254	.16435	2.8845	0.1163	-35.211	1100.565
.12073	53161.781	.16510	2.8841	0.1159	-33.743	1113.142
.12102	53404.367	.16585	2.8838	0.1155	-32.281	1125.724
.12130	53647.000	.16661	2.8834	0.1151	-30.823	1138.309

.12159	53889.684	.16736	2.8830	0.1147	-29.370	1150.898
.12187	54132.418	.16811	2:8826	0.1143	-27.922	1163.490
.12215	54375.203	.16887	2.8822	0.1139	-26.478	1176.087
.12244	54618.035	.16962	2.8818	0.1135	-25.039	1188.687
.12272	54860.914	.17038	2.8814	0.1131	-23.605	1201.291
.12300	55103.844	.17113	2.8811	0.1127	-22.175	1213.898
.12328	55346.824	.17188	2.8807	0.1124	-20.749	1226.510
.12356	55589.852	.17264	2.8803	0.1120	-19.328	1239.125
.12384	55832.922	.17339	2.8799	0.1116	-17.911	1251.743
.12412	56076.039	.17415	2.8795	0.1112	-16.498	1264.366
.12440	56319.207	.17490	2.8792	0.1109	-15.090	1276.992
.12468	56562.422	.17566	2.8788	0.1105	-13.686	1289.621
.12495	56805.680	.17642	2.8784	0.1102	-12.286	1302.255
.12523	57048.984	.17717	2.8780	0.1098	-10.890	1314.891
.12551	57292.332	.17793	2.8777	0.1095	-9.498	1327.532
.12578	57535.723	.17868	2.8773	0.1092	-8.110	1340.176
.12606	57779.164	.17944	2.8769	0.1088	-6.726	1352.823
.12633	58022.648	.18019	2.8765	0.1085	-5.345	1365.474
.12661	58266.176	.18095	2.8762	0.1082	-3.969	1378.129
.12688	58509.746	.18171	2.8758	0.1078	-2.596	1390.787
.12716	58753.359	.18246	2.8754	0.1075	-1.227	1403.449

ANALYSIS PROGRAM OUTPUT

$ TYPE AD122015.DAT ··

DE ELONG	DE TEN	DE TAU	DE DIA	DE ANGLE	DE DEPTH	DE REACH
.05464	31721.846	.09852	2.9769	0.2182	-200.000	0.000
.05485	31949.996	.09922	2.9766	0.2155	-197.434	11.639
.05508	32178.420	.09993	2.9763	0.2130	-194.899	23.288
.05531	32407.109	.10064	2.9760	0.2105	-192.392	34.945
.05554	32636.049	.10135	2.9756	0.2081	-189.913	46.611
.05576	32865.234	.10207	2.9753	0.2057	-187.461	58.285
.05599	33094.652	.10278	2.9750	0.2035	-185.035	69.968
.05622	33324.297	.10349	2.9746	0.2013	-182.636	81.658
.05644	33554.160	.10421	2.9743	0.1991	-180.261	93.356
.05667	33784.238	.10492	2.9740	0.1971	-177.910	105.061
.05690	34014.512	.10564	2.9737	0.1950	-175.582	116.774
.05712	34244.980	.10635	2.9733	0.1931	-173.278	128.494
.05735	34475.645	.10707	2.9730	0.1912	-170.995	140.221
.05757	34706.496	.10778	2.9727	0.1893	-168.735	151.954
.05779	34937.523	.10850	2.9724	0.1875	-166.495	163.695
.05802	35168.727	.10922	2.9720	0.1858	-164.276	175.441
.05824	35400.098	.10994	2.9717	0.1840	-162.076	187.194
.05846	35631.629	.11066	2.-9714	0.1824	-159.897	198.954
.05869	35863.320	.11138	2.9711	0.1807	-157.736	210.719
.05891	36095.164	.11210	2.9708	0.1792	-155.594	222.490
.05913	36327.156	.11282	2.9704	0.1776	-153.469	234.267
.05935	36559.293	.11354	2.9701	0.1761	-151.363	246.050
.05957	36791.574	.11426	2.9698	0.1746	-149.273	257.838
.05979	37023.988	.11498	2.9695	0.1732	-147.200	269.632
.06001	37256.535	.11570	2.9692	0.1718	-145.144	281.431
.06023	37489.211	.11643	2.9688	0.1704	-143.104	293.235
.06045	37722.016	.11715	2.9685	0.1691	-141.079	305.045
.06066	37954.945	.11787	2.9682	0.1677	-139.070	316.860
.06088	38187.996	.11860	2.9679	0.1665	-137.075	328.679
.06110	38421.164	.11932	2.9676	0.1652	-135.095	340.504
.06132	38654.445	.12004	2.9673	0.1640	-133.130	352.334
.06153	38887.844	.12077	2.9670	0.1628	-131.178	364.168
.06175	39121.348	.12149	2.9667	0.1616	-129.240	376.007
.06196	39354.961	.12222	2.9663	0.1605	-127.316	387.851
.06218	39588.676	.12295	2.9660	0.1593	-125.405	399.699
.06239	39822.496	.12367	2.9657	0.1582	-123.506	411.552
.06261	40056.410	.12440	2.9654	0.1571	-121.620	423.409
.06282	40290.426	.12513	2.9651	0.1561	-119.746	435.271
.06304	40524.535	.12585	2.9648	0.1550	-117.885	447.137
.06325	40758.745	.12658	2.9645	0.1540	-116.035	459.008
.06346	40993.043	.12731	2.9642	0.1530	-114.197	470.882
.06367	41227.430	.12804	2.9639	0.1521	-112.370	482.761
.06388	41461.906	.12876	2.9636	0.1511	-110.554	494.644
.06410	41696.469	.12949	2.9633	0.1502	-108.750	506.531
.06431	41931.117	.13022	2.9630	0.1492	-106.956	518.422
.06452	42165.848	.13095	2.9627	0.1483	-105.172	530.317
.06473	42400.660	.13168	2.9624	0.1474	-103.399	542.216

ANALYSIS PROGRAM OUTPUT

.06494	42635.555	.13241	2.9621	0.1466	-101.636	554.119
.06515	42870.527	.13314	2.9618	0.1457	-99.883	566.026
.06535	43105.578	.13387	2.9615	0.1449	-98.140	577.936
.06556	43340.707	.13460	2.9612	0.1440	-96.406	589.850
.06577	43575.906	.13533	2.9609	0.1432	-94.682	601.768
.06598	43811.180	.13606	2.9606	0.1424	-92.967	613.690
.06619	44046.527	.13679	2.9603	0.1417	-91.261	625.616
.06639	44281.945	.13752	2.9600	0.1409	-89.564	637.545
.06660	44517.434	.13825	2.9597	0.1401	-87.876	649.478
.06680	44752.992	.13898	2.9594	0.1394	-86.196	661.414
.06701	44988.617	.13972	2.9591	0.1387	-84.525	673.354
.06721	45224.305	.14045	2.9588	0.1379	-82.863	685.297
.06742	45460.063	.14118	2.9585	0.1372	-81.208	697.244
.06762	45695.883	.14191	2.9582	0.1365	-79.562	709.194
.06783	45931.766	.14265	2.9579	0.1358	-77.923	721.147
.06803	46167.719	.14338	2.9576	0.1352	-76.293	733.104
.06824	46403.727	.14411	2.9574	0.1345	-74.670	745.065
.06844	46639.801	.14484	2.9571	0.1339	-73.054	757.029
.06864	46875.934	.14558	2.9568	0.1332	-71.446	768.996
.06884	47112.125	.14631	2.9565	0.1326	-69.846	780.966
.06904	47348.375	.14704	2.9562	0.1320	-68.252	792.940
.06925	47584.684	.14778	2.9559	0.1313	-66.666	804.916
.06945	47821.047	.14851	2.9556	0.1307	-65.087	816.896
.06965	48057.469	.14925	2.9553	0.1301	-63.514	828.880
.06985	48293.941	.14998	2.9550	0.1296	-61.948	840.866
.07005	48530.477	.15072	2.9548	0.1290	-60.389	852.856
.07025	48767.063	.15145	2.9545	0.1284	-58.837	864.849
.07045	49003.699	.15219	2.9542	0.1279	-57.291	876.844
.07065	49240.395	.15292	2.9539	0.1273	-55.752	888.843
.07084	49477.137	.15366	2.9536	0.1268	-54.218	900.345
.07104	49713.934	.15439	2.9533	0.1262	-52.691	912.850
.07124	49950.781	.15513	2.9531	0.1257	-51.170	924.858
.07144	50187.676	.15586	2.9528	0.1252	-49.655	936.869
.07163	50424.621	.15660	2.9525	0.1247	-48.146	948.884
.07183	50661.621	.15733	2.9522	0.1242	-46.643	960.901
.07203	50898.668	.15807	2.9519	0.1237	-45.146	972.921
.07222	51135.762	.15881	2.9517	0.1232	-43.654	984.944
.07242	51372.898	.15954	2.9514	0.1227	-42.168	996.969
.07262	51610.086	.16028	2.9511	0.1222	-40.688	1008.998
.07281	51847.320	.16102	2.9508	0.1217	-39.213	1021.030
.07301	52084.602	.16175	2.9505	0.1213	-37.743	1033.065
.07320	52321.926	.16249	2.9503	0.1208	-36.279	1045.102
.07339	52559.297	.16323	2.9500	0.1203	-34.820	1057.142
.07359	52796.711	.16396	2.9497	0.1199	-33.366	1069.185
.07378	53034.172	.16470	2.9494	0.1195	-31.917	1081.231
.07397	53271.680	.16544	2.9492	0.1190	-30.473	1093.280
.07417	53509.230	.16618	2.9489	0.1186	-29.035	1105.332
.07436	53746.824	.16692	2.9486	0.1182	-27.601	1117.386
.07455	53984.457	.16765	2.9483	0.1177	-26.172	1129.443
.07474	54222.133	.16839	2.9481	0.1173	-24.748	1141.503
.07493	54459.852	.16913	2.9478	0.1169	-23.328	1153.565

.07513	54697.609	.16987	2.9475	0.1165	-21.913	1165.631
.07532	54935.410	.17061	2.9473	0.1161	-20.503	1177.699
.07551	55173.250	.17135	2.9470	0.1157	-19.098	1189.769
.07570	55411.133	.17208	2.9467	0.1153	-17.697	1201.843
.07589	55649.055	.17282	2.9464	0.1149	-16.300	1213.919
.07608	55887.016	.17356	2.9462	0.1146	-14.908	1225.998
.07627	56125.016	.17430	2.9459	0.1142	-13.520	1238.079
.07646	56363.055	.17504	2.9456	0.1138	-12.136	1250.163
.07664	56601.133	.17578	2.9454	0.1134	-10.757	1262.250
.07683	56839.250	.17652	2.9451	0.1131	-9.382	1274.339
.07702	57077.406	.17726	2.9448	0.1127	-8.010	1286.431
.07721	57315.602	.17800	2.9446	0.1124	-6.643	1298.525
.07740	57553.832	.17874	2.9443	0.1120	-5.280	1310.622
.07758	57792.102	.17948	2.9440	0.1117	-3.921	1322.722
.07777	58030.406	.18022	2.9438	0.1113	-2.566	1334.824
.07796	58268.750	.18096	2.9435	0.1110	-1.215	1346.929

ANALYSIS PROGRAM OUTPUT

WET DESIGN DATA

DRY DESIGN DATA

15	30970	200
1140	0.24	0.1
97	1	
3.25	322000	
0.099		
0.183	0.182	

VN	VT	RN	FN	FT	X	VN	VT	RN	FN	FT	X
3.084	14.679	86300	0	0	0	3.246	14.644	92500	0	0	0
3.049	14.687	85300	7.989	23.241	12.084	3.208	14.653	91400	8.695	22.745	11.639
3.014	14.694	84300	7.81	23.267	24.178	3.17	14.661	90300	8.494	22.772	23.288
2.981	14.701	83400	7.638	23.293	36.281	3.134	14.669	89300	8.3	22.799	34.945
2.948	14.707	82500	7.474	23.317	48.394	3.099	14.676	88300	8.115	22.825	46.611
2.917	14.714	81600	7.316	23.341	60.516	3.064	14.684	87300	7.937	22.85	58.285
2.886	14.72	80700	7.164	23.363	72.646	3.031	14.691	86300	7.766	22.874	69.968
2.856	14.726	79900	7.017	23.386	84.786	2.999	14.697	85400	7.602	22.897	81.658
2.827	14.731	79000	6.877	23.407	96.934	2.967	14.704	84500	7.444	22.919	93.356
2.799	14.736	78200	6.741	23.428	109.091	2.937	14.71	83600	7.293	22.941	105.061
2.772	14.742	77500	6.611	23.448	121.255	2.907	14.716	82800	7.146	22.962	116.774
2.745	14.747	76700	6.485	23.467	133.428	2.878	14.721	81900	7.006	22.982	128.494
2.719	14.751	76000	6.364	23.486	145.608	2.85	14.727	81100	6.87	23.001	140.221
2.694	14.756	75200	6.247	23.504	157.797	2.823	14.732	80300	6.74	23.02	151.954
2.67	14.761	74500	6.134	23.522	169.992	2.796	14.737	79600	6.614	23.039	163.695
2.646	14.765	73900	6.025	23.539	182.196	2.77	14.742	78800	6.492	23.056	175.441
2.622	14.769	73200	5.92	23.556	194.406	2.745	14.747	78100	6.374	23.073	187.194
2.599	14.773	72600	5.818	23.572	206.624	2.72	14.751	77400	6.261	23.09	198.954
2.577	14.777	71900	5.719	23.588	218.848	2.696	14.756	76700	6.151	23.106	210.719
2.555	14.781	71300	5.624	23.604	231.08	2.673	14.76	76000	6.045	23.122	222.49
2.534	14.784	70700	5.532	23.619	243.318	2.65	14.764	75400	5.943	23.137	234.267
2.514	14.788	70100	5.442	23.634	255.564	2.628	14.768	74700	5.844	23.152	246.05
2.493	14.791	69500	5.356	23.648	267.815	2.606	14.772	74100	5.748	23.167	257.838
2.474	14.795	69000	5.272	23.662	280.074	2.585	14.776	73500	5.654	23.181	269.632
2.454	14.798	68400	5.191	23.676	292.338	2.564	14.779	72900	5.564	23.195	281.431
2.435	14.801	67900	5.112	23.689	304.609	2.544	14.783	72300	5.477	23.208	293.235
2.417	14.804	67400	5.036	23.702	316.886	2.524	14.786	71700	5.392	23.221	305.045
2.399	14.807	66900	4.961	23.715	329.17	2.504	14.789	71200	5.31	23.234	316.86
2.381	14.81	66300	4.889	23.728	341.459	2.485	14.793	70600	5.23	23.246	328.679
2.364	14.813	65900	4.819	23.74	353.755	2.467	14.796	70100	5.153	23.258	340.504
2.347	14.815	65400	4.751	23.752	366.056	2.449	14.799	69600	5.078	23.27	352.334
2.331	14.818	64900	4.685	23.764	378.363	2.431	14.802	69100	5.004	23.282	364.168
2.314	14.82	64400	4.621	23.775	390.676	2.413	14.805	68600	4.933	23.293	376.007
2.299	14.823	64000	4.558	23.786	402.995	2.396	14.807	68100	4.864	23.304	387.851
2.283	14.825	63600	4.497	23.797	415.319	2.38	14.81	67600	4.797	23.315	399.699
2.268	14.828	63100	4.438	23.808	427.649	2.363	14.813	67100	4.732	23.325	411.552
2.253	14.83	62700	4.381	23.819	439.984	2.347	14.815	66600	4.669	23.336	423.409
2.238	14.832	62300	4.324	23.829	452.325	2.332	14.818	66200	4.607	23.346	435.271
2.224	14.834	61900	4.27	23.839	464.671	2.316	14.82	65800	4.547	23.356	447.137
2.21	14.836	61500	4.216	23.85	477.023	2.301	14.822	65300	4.488	23.365	459.008
2.196	14.838	61100	4.165	23.859	489.38	2.287	14.825	64900	4.431	23.375	470.882
2.183	14.84	60700	4.114	23.869	501.742	2.272	14.827	64500	4.375	23.384	482.761

ANALYSIS PROGRAM OUTPUT

2.169	14.842	60300	4.065	23.879	514.11	2.258	14.829	64100	4.321	23.393	494.644
2.156	14.844	59900	4.017	23.888	526.482	2.244	14.831	63700	4.268	23.402	506.531
2.144	14.846	59600	3.97	23.897	538.86	2.23	14.833	63300	4.217	23.411	518.422
2.131	14.848	59200	3.924	23.906	551.242	2.217	14.835	62900	4.166	23.42	530.317
2.119	14.85	58900	3.879	23.915	563.63	2.204	14.837	62500	4.117	23.428	542.216
2.107	14.851	58500	3.835	23.924	576.023	2.191	14.839	62100	4.069	23.436	554.119
2.095	14.853	58200	3.793	23.933	588.42	2.178	14.841	61800	4.023	23.444	566.026
2.083	14.855	57900	3.751	23.941	600.823	2.165	14.843	61400	3.977	23.452	577.936
2.072	14.856	57500	3.71	23.949	613.23	2.153	14.845	61000	3.932	23.46	589.85
2.06	14.858	57200	3.671	23.958	625.642	2.141	14.846	60700	3.889	23.468	601.768
2.049	14.859	56900	3.632	23.966	638.059	2.129	14.848	60400	3.846	23.476	613.69
2.039	14.861	56600	3.594	23.974	650.481	2.118	14.85	60000	3.805	23.483	625.616
2.028	14.862	56300	3.556	23.982	662.907	2.106	14.851	59700	3.764	23.491	637.545
2.017	14.864	56000	3.52	23.989	675.338	2.095	14.853	59400	3.724	23.498	649.478
2.007	14.865	55700	3.485	23.997	687.774	2.084	14.855	59000	3.685	23.505	661.414
1.997	14.866	55400	3.45	24.005	700.214	2.073	14.856	58700	3.647	23.512	673.354
1.987	14.868	55100	3.416	24.012	712.659	2.062	14.858	58400	3.61	23.519	685.297
1.977	14.869	54800	3.382	24.02	725.108	2.052	14.859	58100	3.574	23.526	697.244
1.967	14.87	54600	3.35	24.027	737.562	2.042	14.86	57800	3.538	23.532	709.194
1.958	14.872	54300	3.318	24.034	750.02	2.031	14.862	57500	3.503	23.539	721.147
1.948	14.873	54000	3.286	24.041	762.483	2.021	14.863	57200	3.469	23.545	733.104
1.939	14.874	53700	3.256	24.048	774.95	2.012	14.865	57000	3.436	23.552	745.065
1.93	14.875	53500	3.226	24.055	787.421	2.002	14.866	56700	3.403	23.558	757.029
1.921	14.876	53200	3.196	24.062	799.897	1.992	14.867	56400	3.371	23.564	768.996
1.912	14.878	53000	3.167	24.069	812.377	1.983	14.868	56100	3.339	23.57	780.966
1.904	14.879	52700	3.139	24.075	824.862	1.974	14.87	55900	3.308	23.577	792.94
1.895	14.88	52500	3.111	24.082	837.351	1.965	14.871	55600	3.278	23.583	804.916
1.887	14.881	52200	3.084	24.088	849.844	1.956	14.872	55300	3.249	23.588	816.896
1.878	14.882	52000	3.057	24.095	862.341	1.947	14.873	55100	3.22	23.594	828.88
1.87	14.883	51800	3.031	24.101	874.843	1.938	14.874	54800	3.191	23.6	840.866
1.862	14.884	51500	3.005	24.107	887.348	1.929	14.875	54600	3.163	23.606	852.856
1.854	14.885	51300	2.98	24.114	899.858	1.921	14.876	54300	3.136	23.611	864.849
1.846	14.886	51100	2.955	24.12	912.372	1.913	14.878	54100	3.109	23.617	876.844
1.839	14.887	50900	2.931	24.126	924.89	1.904	14.879	53900	3.083	23.622	888.843
1.831	14.888	50700	2.907	24.132	937.412	1.896	14.88	53600	3.057	23.628	900.845
1.824	14.889	50400	2.884	24.138	949.939	1.888	14.881	53400	3.031	23.633	912.85
1.816	14.89	50200	2.861	24.144	962.469	1.881	14.882	53200	3.006	23.638	924.858
1.809	14.891	50000	2.838	24.15	975.003	1.873	14.883	52900	2.982	23.644	936.869
1.802	14.891	49800	2.816	24.156	987.542	1.865	14.884	52700	2.958	23.649	948.884
1.795	14.892	49600	2.794	24.161	1000.084	1.858	14.885	52500	2.934	23.654	960.901
1.788	14.893	49400	2.773	24.167	1012.63	1.85	14.885	52300	2.911	23.659	972.921
1.781	14.894	49200	2.752	24.173	1025.181	1.843	14.886	52100	2.888	23.664	984.944
1.774	14.895	49000	2.731	24.178	1037.735	1.836	14.887	51900	2.866	23.669	996.969
1.767	14.896	48800	2.711	24.184	1050.293	1.828	14.888	51700	2.844	23.674	1008.998
1.761	14.896	48600	2.691	24.189	1062.855	1.821	14.889	51500	2.822	23.678	1021.03
1.754	14.897	48500	2.671	24.195	1075.421	1.814	14.89	51300	2.801	23.683	1033.065
1.748	14.898	48300	2.652	24.2	1087.991	1.808	14.891	51100	2.78	23.688	1045.102
1.741	14.899	48100	2.633	24.205	1100.565	1.801	14.892	50900	2.759	23.693	1057.142
1.735	14.899	47900	2.614	24.211	1113.142	1.794	14.892	50700	2.739	23.697	1069.185
1.729	14.9	47700	2.596	24.216	1125.724	1.788	14.893	50500	2.719	23.702	1081.231
1.723	14.901	47600	2.578	24.221	1138.309	1.781	14.894	50300	2.7	23.706	1093.28

1.717	14.901	47400	2.56	24.226	1150.898	1.775	14.895	50100	2.68	23.711	1105.332
1.711	14.902	47200	2.542	24.232	1163.49	1.768	14.895	49900	2.661	23.715	1117.386
1.705	14.903	47000	2.525	24.237	1176.087	1.762	14.896	49700	2.643	23.72	1129.443
1.699	14.903	46900	2.508	24.242	1188.687	1.756	14.897	49600	2.624	23.724	1141.503
1.693	14.904	46700	2.492	24.247	1201.291	1.75	14.898	49400	2.606	23.728	1153.565
1.687	14.905	46500	2.475	24.252	1213.898	1.744	14.898	49200	2.589	23.733	1165.631
1.682	14.905	46400	2.459	24.257	1226.51	1.738	14.899	49000	2.571	23.737	1177.699
1.676	14.906	46200	2.443	24.261	1239.125	1.732	14.9	48900	2.554	23.741	1189.769
1.671	14.907	46100	2.427	24.266	1251.743	1.726	14.9	48700	2.537	23.745	1201.843
1.665	14.907	45900	2.412	24.271	1264.366	1.72	14.901	48500	2.52	23.75	1213.919
1.66	14.908	45800	2.397	24.276	1276.992	1.714	14.902	48400	2.504	23.754	1225.998
1.655	14.908	45600	2.382	24.281	1289.621	1.709	14.902	48200	2.488	23.758	1238.079
1.649	14.909	45500	2.367	24.285	1302.255	1.703	14.903	48000	2.472	23.762	1250.163
1.644	14.91	45300	2.352	24.29	1314.891	1.698	14.904	47900	2.456	23.766	1262.25
1.639	14.91	45200	2.338	24.295	1327.532	1.692	14.904	47700	2.441	23.77	1274.339
1.634	14.911	45000	2.324	24.299	1340.176	1.687	14.905	47600	2.425	23.774	1286.431
1.629	14.911	44900	2.31	24.304	1352.823	1.682	14.905	47400	2.41	23.778	1298.525
1.624	14.912	44700	2.296	24.308	1365.474	1.677	14.906	47300	2.396	23.782	1310.622
1.619	14.912	44600	2.283	24.313	1378.129	1.671	14.907	47100	2.381	23.785	1322.722
1.614	14.913	44500	2.269	24.317	1390.787	1.666	14.907	47000	2.367	23.789	1334.824
1.61	14.913	44300	2.256	24.322	1403.449	1.661	14.908	46800	2.352	23.793	1346.929

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